# Full-Duplex or Half-Duplex: A Bayesian Game for Wireless Networks with Heterogeneous Self-Interference Cancellation Capabilities 

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#### Abstract

Recently, tremendous progress has been made in self-interference cancellation (SIC) techniques that enable a wireless device to transmit and receive data simultaneously on the same frequency channel, a.k.a. in-band full-duplex (FD). Although operating in FD mode significantly improves the throughput of a single wireless link, it doubles the number of concurrent transmissions, which limits the potential for coexistence between multiple FD-enabled links. In this paper, we consider the coexistence problem of concurrent transmissions between multiple FD-enabled links with different SIC capabilities; each link can operate in either FD or half-duplex mode. First, we consider two links and formulate the interactions between them as a Bayesian game. In this game, each link tries to maximize its throughput while minimizing the transmission power cost. We derive a closed-form expression for the Bayesian Nash equilibrium and determine the conditions under which no outage occurs at either link. Then, we study the coexistence problem between more than two links, assuming that each link is only affected by its dominant interfering link. We show that under this assumption, no more than two links will be involved in a single game. Finally, we corroborate our analytical findings via extensive simulations and numerical results.


Index Terms-In-band full-duplex, self-interference cancellation, coexistence, Bayesian game, full-duplex wireless networks

## 1 Introduction

CLASSICAL wireless systems achieve bidirectional communications by separating the forward and reverse links in time, i.e., time division duplexing (TDD), or frequency, i.e., frequency division duplexing (FDD). Some wireless systems, such as 4G LTE, support both schemes (e.g., LTE-TDD and LTE-FDD). The challenge of achieving simultaneous transmission and reception on the same frequency channel, i.e., inband full-duplex (FD), is related to the strong self-interference that arises when a device that is receiving an information signal attempts to transmit another signal at the same time [2]. Because of path loss, the received power of the intended signal from the peer node is often much weaker than the node's self-interference. This results in saturating the analog-to-digital converter (ADC) and prevents packet decoding.

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Recently, new designs for analog and digital self-interference cancellation (SIC) techniques have been proposed (see [3], [4] for a survey), which together provide up to 110 dB SIC on a single-antenna FD transceiver [5].

From one link's perspective, the advantage of in-band FD communications is clear; it basically doubles the link's throughput. However, such gain is less obvious in the case of a network of multiple interfering links. When these links operate in the same vicinity (i.e., the same collision domain), it is not always optimal for all links to operate in FD mode [4]. To illustrate, consider the three scenarios in Fig. 1. In each scenario, two links are active at the same time and over the same frequency channel. According to the scenario in Fig. 1a, transmitting in an HD fashion enables both links $a \rightarrow b$ and $c \rightarrow d$ to operate simultaneously over the same channel, achieving a total throughput of $2 R$ bps (For simplicity, in this example we assume that all transmissions are associated with a constant rate $R$ ). However, if link $a \rightarrow b$ switches unilaterally from HD to FD, as shown in Fig. 1b, collisions may occur at both nodes $a$ and $d$, reducing the network throughput to $R$ bps (only $a \rightarrow b$ transmission is successful). One reason for the collision at node $a$ is that the SINR at node $a$ becomes lower than the SINR threshold (which is function of the modulation and coding scheme used) needed for successful decoding of the message from node $b$. Under the assumed path-loss model, the occurrence of this scenario depends mainly on the distances between nodes $a$ and $b$, and between nodes $a$ and $c$.

If link $c \rightarrow d$ switches to FD mode instead of $a \rightarrow b$, the same argument used in the previous case applies. If both links operate in FD mode, collisions will occur at all four


Fig. 1. Implications of operating wireless networks in FD fashion. (a) (HD, HD) strategy achieves a total throughput of $2 R \mathrm{bps}$, (b) (FD, HD) strategy achieves $R$ bps, while (c) (FD, FD) strategy achieves zero throughput. The arcs represent the transmission ranges of different nodes.
nodes, reducing the network throughput to zero (see Fig. 1c). Note that this is a simplified example of a small network with only two links. The situation worsens with more interfering links.

Most existing literature that studies the performance of FD networks assume perfect SIC (A detailed discussion of the related work is given in Section 2). In [6] the authors proposed stochastic resource allocation schemes for LTE networks with heterogeneous (perfect and imperfect) SIC capabilities. These schemes are centralized and exhibit high computational complexity. In this paper, we develop a game-theoretic framework to analyze the performance of wireless networks with heterogeneous SIC capabilities and in the absence of any global information, i.e., a link does not know the exact SIC capability of its neighboring link. The key idea is to allow nodes to determine, in a distributed fashion, the optimal operation mode (FD or HD). Such a distributed approach is critical to many wireless networks, including heterogeneous networks that coexist on the same band, e.g., LTE and Wi-Fi that operate on the unlicensed 5 GHz bands [7]. In these scenarios, it is unlikely to have collaboration between the Wi-Fi AP and the LTE-U/ LAA HeNB such that they know the exact SIC of each other. This applies even if the network is small (e.g., coexistence between a single Wi-Fi link and a single LTE-U/LAA link).

The contributions of this paper can be summarized as follows:
(1) We analyze a simple (HD versus FD) game between two bidirectional links to gain insight into the coexistence problem. In this game, links (players) know the exact SIC capabilities of each other. Simple utilities and interference models are considered. We find that the outcome of this game depends on the residual self-interference (RSI) (due to imperfect SIC) and external interference (from one link on the other).
(2) To capture the heterogeneity in SIC capabilities, we then formulate a Bayesian game between the two links. In this game, the SIC capability of each link specifies its 'type,' which is unknown to the other link. We derive the Bayesian Nash equilibrium (BNE) for this game. From our analysis, we observe that the range of possible SIC values a given player may take can be divided into three regions (types). In two types, either the HD or the FD strategy is dominant, while the BNE in the third type depends mainly on the probability distribution of the other
player's types. The thresholds that specify the regions of various SIC types depend, among other factors, on the outage probabilities of the player's forward and reverse links. Accordingly, we derive closed-form expressions for these thresholds under different outage scenarios.
(3) To capture the interactions between more than two links, we consider a network of FD-enabled links with heterogeneous SIC capabilities and study the coexistence problem. Analyzing the Bayesian game for more than two links is very complex and intractable due to the mutual dependence between players' actions and the fact that the exact SICs of players are unknown. Hence, we study the multi-player Bayesian game under the simplifying assumption that every player is mainly affected by a single dominating link. Finally, we corroborate our analytical findings via extensive simulations and numerical results.
The rest of the paper is organized as follows. We discuss some related works in Section 2 and describe the simple two-player strategic game in Section 3. The Bayesian game is formulated and analyzed in Section 4. In Section 5, we extend the two-player Bayesian game to more than two links. The proposed games are simulated in Section 6. Finally, in Section 7 we conclude the paper and provide directions for future research.

## 2 Related Work

There are several efforts in the literature that studied and demonstrated the feasibility of FD communications from one link's perspective [5], [8], [9], [10], [11], [12], [13]. The feasibility of network-wide FD communications was discussed in [14], [15], [16], [17], [18], [19], [20], [21]. In [14], the authors analyzed the outage probability for FD-based cellular systems based on two schemes: Bidirectional FD communications between the base station (BS) and the user equipment (UE) and three-node scheme (the BS transmits to a UE while simultaneously receiving from another UE). The authors found that the three-node scheme has better performance and is more practical than the two-node scheme if the RSI signal is relatively high. The authors in [15] theoretically studied the benefits of FD communications for multihop networks by computing the achievable end-to-end throughput. They showed that in some scenarios, the benefits of FD communications in multi-hop networks can exceed double that of HD networks.

In [16] the authors studied the multiplexing gain offered by FD communications in multi-cell networks. In addition to RSI, the authors considered the interference from uplink nodes on the reception of the downlink nodes (assuming a three-node setting). To remedy this interference, the authors considered a solution based on spatial interference alignment. In [17], the authors proposed an analytical framework to analyze the capacity gain of FD networks, assuming a CSMA-based MAC protocol. They found that spatial reuse and inter-link interference may cause significant reduction to network capacity. To enable FD Wi-Fi networks, the authors in [18] proposed a MAC protocol and presented an experimental and simulation study to assess the performance of their scheme. In contrast to [17], they found that FD communications can potentially enhance the performance
of Wi-Fi networks. The authors in [20] considered the internode interference in a three-node (base station, downlink node, and uplink node) FD OFDM network. To maximize the network sum-throughput for this scenario, they proposed different power allocation schemes while taking into account inter-node interference.

Game theory has been used to study FD wireless networks [22], [23], [24], [25], [26]. In [22], the authors discussed several game theoretic models, including evolutionary games used for adaptive mode switching problems, auc-tion-based models used for power allocation problems, and matching theory used for subcarrier allocation. For distributed FD networks, the authors discussed non-cooperative games used for power control, matching games used for opportunistic channel assignment, and coalition games used for collaborative spectrum sensing. In [23], the authors considered virtualization in FD relay cellular networks, where they used a three-stage Stackelberg game to model a resource management problem. In [24], the authors proposed a cooperative game to solve the hidden-node problem in FD-enabled cognitive radio networks. Specifically, they exploited the possibility of simultaneous transmission and sensing on the same channel using SIC techniques. In [26], the authors proposed an adaptive FD/HD mode switching based on the residual self-interference and channel conditions to maximize the ergodic capacity. The above mentioned game-theoretic studies did not account for the heterogeneity and imperfectness in the SIC capabilities.

The achievable throughput of FD MIMO networks was investigated in [19], [21]. In [19] the authors considered a binary interference model of a general network topology and derived the conditions under which different technologies (MIMO, multi-user MIMO, and FD communications) achieve the best performance. In [21] the authors considered the problem of power minimization in an FD MIMO network subject to rate demands. They derived conditions under which the FD mode outperforms the HD mode. In [27], the authors considered MIMO FD relay systems, and specifically amplify-and-forward (AF) cooperative networks. They proposed a joint precoding/decoding system that maximizes the end-to-end system performance while reducing complexity. In [28], the authors proposed a joint transceiver design algorithm for the FD $k$-pair MIMO interference channel with simultaneous wireless information and power transfer. Using optimization theory, they analyzed two problems: The sum-power minimization problem and the sum-rate maximization problem. In [29], [30], [31], [32], the authors investigated the incorporation of SIC techniques in opportunistic spectrum access (OSA) systems. In overlay models, secondary users (SUs) operate in either FD mode or simultaneous transmit-sense mode. On the other hand, in an underlay OSA network, SUs operate in either HD or FD mode. The authors in [29], [30], [31], [32] determined the optimal operating mode for both overlay and underlay settings. Our work is different since it has different objectives and follows a different approach.

In [33], the authors proposed several stochastic formulations to minimize the cost of composing a virtual LTE-U network from a hybrid set of HD/FD Wi-Fi access points (APs). An AP is assumed to be either HD or FD (with perfect SIC). In general, different devices in a network may
have different SIC capabilities, mainly depending on the employed analog/digital cancellation mechanisms. In [6] the authors proposed joint channel and BS stochastic allocation schemes for opportunistic LTE networks with heterogeneous SIC capabilities at different BSs. The resource allocation schemes in [6], [33] are centralized with high computational complexity. Instead, in this paper we develop a game-theoretic framework to analyze the performance of wireless networks with heterogeneous SIC capabilities and in the absence of any global information (i.e., a link does not know the exact SIC level of its neighboring link).

Stochastic geometry has been used in the literature to model FD-enabled wireless networks and assess their performance [34], [35]. The authors in [34] developed a spatial stochastic framework to study the tradeoff between network throughput (i.e., FD operation) and spatial reuse for CSMAbased networks. They analyzed the FD gain as a function of link distance, interference range, network density, and carrier sensing schemes. In [35] the authors studied the network capacity under the FD mode using stochastic geometry. They found that the aggregated interference is the main challenge for scalable FD networks. A unique feature of our scheme is the consideration of imperfect SIC, where nodes are unaware of other nodes SIC capability, which directly affects the optimal operation mode. In [1], we studied the coexistence problem between two FD-enabled wireless links with heterogeneous SIC capabilities. We formulated the problem as a two-player Bayesian game and analyzed the performance at the BNE. In this paper, we extend our study and simulations to more than two coexisting FD-enabled links.

## 3 Simple Normal-Form HD versus FD Game

In this section, we study a simple strategic game between two coexisting links to identify their stable operating modes. For now, we assume that the two players (links) know all the parameters of the game, including the SIC capability of the other player. Furthermore, we consider simple utility functions and interference models. The game can be defined in the strategic form as $\mathcal{G}_{s}=(\mathcal{P}, \mathcal{S}, \mathcal{U})$, where $\mathcal{P}=\left\{P_{1}, P_{2}\right\}$ is the set of players, $\mathcal{S}=\left\{S_{1}, S_{2}\right\}$ is the set of strategies, and $\mathcal{U}=\left\{U_{1}, U_{2}\right\}$ is the set of utilities for the two players. For $i=1,2$, the strategy set $S_{i}=\{\mathrm{FD}, \mathrm{HD}\}$ is the set of pure strategies of $P_{i}$. Let $a$ and $b$ be the two nodes associated with $P_{1}$, and let $c$ and $d$ be the two nodes associated with $P_{2}$.

An FD strategy of a given player $P_{i}$ indicates that both nodes of that player/link will transmit simultaneously on the same channel. On the other hand, under an HD strategy, only a single node of player $P_{i}$ will transmit data to its peer. For simplicity, we assume that under the HD strategy, the transmitter are nodes $a$ and $c$ in case of $P_{1}$ and $P_{2}$, respectively. In practice, there will be a contention phase before the game starts, where each link decides the direction of the HD communications in the game (e.g., a to b or b to a for $P_{1}$ and similarly for $P_{2}$ ). To capture this contention phase, we designate the nodes of each link as master and slave. A master node is the initiator of the communications. Based on the MAC-queue lengths at both nodes, the master node decides whether communication under the HD strategy will be in the forward or reverse direction. The slave node

TABLE 1
Utilities for the Simple Game

| $P_{2}$ | FD | HD |
| :--- | :---: | :---: |
| $P_{1}$ | $(0,0)$ | $(2 R, 0)$ |
| FD | $(0,2 R)$ | $(R, R)$ |
| HD | FD | HD |
| (a) High external and self-interference |  |  |
|  | $P_{2}$ | $(2 R, 2 R)$ |
| $P_{1}$ | $(R, 2 R)$ | $(2 R, R)$ |
| FD |  | $(R, R)$ |
| HD |  |  |

(b) Low external interference and perfect SIC
can piggyback the value of the queue length in the ACK packet to help the master node in the selection of the communications direction.

The utilities $\mathcal{U}=\left\{U_{1}, U_{2}\right\}$ of the two links represent their respective throughputs under given strategies and hence their operating modes at the Nash equilibrium (NE), depend on two factors: External interference from one link on the other and residual self-interference. The amount of external interference depends on the channel gains and transmission powers, whereas self-interference is function of the SIC capabilities. Let $R$ be the throughput of a link in one direction when the communication is successful. Table 1 shows two instances of the game $\mathcal{G}_{s}$. In the first instance (Table 1a), both external and self-interference are assumed to be high. As a result, if both $P_{1}$ and $P_{2}$ operate in the FD mode, then their utilities will be zero. Although (FD, FD) is a NE, it is not an efficient one. Moreover, (FD, FD) is not a unique NE. Specifically, (FD, HD) and (HD, FD) are also NEs and, in fact, they are Pareto-efficient. In another instance of $\mathcal{G}_{s}$ (shown in Table 1b), the external interference is low and all nodes have perfect SIC. In this case, (FD, FD) is an efficient and unique NE.

The above game illustrates that FD communications is not always optimal from a network's perspective. To provide a complete characterization of the stable strategy profiles for coexisting FD-enabled links, we account for nodes' SIC capabilities as well as the amount of induced interference from one link on the other in the following Bayesian game.

## 4 Bayesian Game between Two Links with Heterogeneous SIC Capabilities

### 4.1 Game Formulation

The two-player Bayesian game can be defined as $\mathcal{G}=(\mathcal{P}, \mathcal{S}, \Theta, \mathcal{U}, \mathcal{D})$, where $\mathcal{P}$ and $\mathcal{S}$ are as in $\mathcal{G}_{s}$. The exact expressions for the utility set $\mathcal{U}=\left\{U_{1}, U_{2}\right\}$ will be defined later. As for $\Theta$ and $\mathcal{D}$, they are defined as follows:
Definition 4.1. $\Theta=\left\{\Theta_{1}, \Theta_{2}\right\}$, where $\Theta_{i}$ is the set of types of $P_{i}$.
$P_{i}^{\prime}$ s type represents its SIC capability, which is characterized by the parameter $\chi_{i} \in[0,1] . \chi_{i}$ is the ratio between the RSI signal and the transmit power. If $\chi_{i}=0$, the corresponding node can completely suppress its self-interference (perfect SIC); otherwise, it can only suppress a fraction $1-\chi_{i}$ of its self-interference. For simplicity, we assume that both nodes of $P_{i}$ have the same $\chi_{i}$. Although the player's SIC
capability (and hence, its type) takes values in the continuous range between 0 and 1 , we later show that the NE strategy of $P_{i}$ will be the same if $P_{i}$ is of any type between 0 and some threshold $\chi_{i}^{*}$. Also, the NE strategy of $P_{i}$ will be the same if $P_{i}$ is of any type between $\chi_{i}^{*}$ and another threshold $\chi_{i}^{* *}$, and it will be the same if $P_{i}$ is of any type between $\chi_{i}^{* *}$ and 1. The two thresholds $\chi_{i}^{*}$ and $\chi_{i}^{* *}$ are derived at the end of this section. Accordingly, we discretize the set of types $\Theta_{i} \triangleq\{1,2,3\}$, and denote the type of player $P_{i}$ by $\theta_{i}$, which can take one of three possible values: $\theta_{i}=1$ (Type 1) if $\chi_{i} \in\left[0, \chi_{i}^{*}\right], \theta_{i}=2$ (type 2) if $\chi_{i} \in\left[\chi_{i}^{*}, \chi_{i}^{* *}\right]$, and $\theta_{i}=3$ (type 3) if $\chi_{i} \in\left[\chi_{i}^{* *}, 1\right]$. Note that $\theta_{1}$ and $\theta_{2}$ are independent.

Definition 4.2. $\mathcal{D}=\left\{D_{1}, D_{2}\right\}$, where $D_{i}$ is a probability distribution over the types of $P_{i}, i=1,2$.

For $i=1,2, P_{i}$ is of type 1 with probability $p_{i 1}$, of type 2 with probability $p_{i 2}$, and of type 3 with probability $p_{i 3}$.

Finally, in Bayesian games, a pure strategy of $P_{i}$ is a map $s_{i}: \Theta_{i} \rightarrow S_{i}$, prescribing an action for each type of $P_{i}$. Strategy spaces, player types, utility functions, and the probability distributions over the types are assumed to be common knowledge to all players. However, a given player knows only its current strategy and type, and does not know the strategies selected by the other player or its true type. In practice, the probability distributions over the types of players can be estimated by collecting statistics about the latest widely implemented SIC techniques in current radio technologies (e.g., Wi-Fi APs, cellular BSs, tablets, etc.).

The motivation for selecting Bayesian games to model this problem is as follows. Although the development of SIC techniques is still in its early stages, it is expected that different radios (e.g., Wi-Fi APs, BSs, TVs, tablets, etc.) will have different SIC capabilities. Achieving perfect SIC may not always be feasible due to imperfections in existing SIC techniques and challenges related to signal reflections, hardware/DSP limitations, etc. Furthermore, assuming that radios know the exact SIC capabilities of neighboring radios is unrealistic. However, knowing the distributions over players' SIC capabilities is more justifiable. Hence, selecting Bayesian games to model this problem, where perfect knowledge about other players' SIC capabilities is unavailable, is well justified.

Next, we analyze the utility $U_{1}$ for $P_{1}$ ( $U_{2}$ can be analyzed similarly). First, consider $U_{1}$ under the (FD, FD) strategy profile. $P_{1}$ benefits from an FD transmission only if no outage/collision occurs at node $a$ or $b$, i.e., when $\gamma_{i j}^{(\mathrm{FF})} \geq \gamma_{i j}^{*}$, $i, j \in\{a, b\}$, where $\gamma_{i j}^{(\mathrm{FF})}$ is the SNR at node $j$ while receiving data from node $i$ under (FD, FD) and $\gamma_{i j}^{*}$ is a given reception threshold. Define $\mathbb{1}_{i j}^{(\mathrm{FF})}$ to be an indicator function for a successful transmission from $i$ to $j$ under (FD, FD). $\mathbb{1}_{i j}^{(\mathrm{FF})}=1$ if $\gamma_{i j}^{(\mathrm{FF})} \geq \gamma_{i j}^{*}$ and 0 otherwise. Denote the transmission cost from node $i$ by $C_{i}$. We set $C_{i}=c_{i} P_{t, i}$, where $P_{t, i}$ is the transmission power of node $i, i \in\{a, b, c, d\}$, and $c_{i}$ is a constant of conversion measured in bits per second per watts. Note that $c_{i}$ is a design parameter that can be used to reflect the relative importance of the transmission power. The utility of $P_{1}$ under (FD, FD), denoted by $U_{1}^{(\mathrm{FF})}$, can be expressed as follows:

$$
\begin{equation*}
U_{1}^{(\mathrm{FF})}=\mathbb{1}_{a b}^{(\mathrm{FF})} R_{a b}^{(\mathrm{FF})}+\mathbb{1}_{b a}^{(\mathrm{FF})} R_{b a}^{(\mathrm{FF})}-C_{a}-C_{b} \tag{1}
\end{equation*}
$$

where $R_{a b}^{(\mathrm{FF})}$ and $R_{b a}^{(\mathrm{FF})}$ are, respectively, the forward and reverse link throughputs for link $(a, b) . R_{i j}^{(\mathrm{FF})}, i, j \in\{a, b\}$, $i \neq j$, is given by:

$$
\begin{align*}
R_{i j}^{(\mathrm{FF})} & =\log \left(1+\gamma_{i j}^{(\mathrm{FF})}\right) \\
& =\log \left(1+\frac{P_{t, i} g_{i j}}{\chi_{j}^{2} P_{t, j} g_{j j}+P_{t, c} g_{c j}+P_{t, d} g_{d j}+\sigma_{j}^{2}}\right) \tag{2}
\end{align*}
$$

where $g_{i j}$ is the channel gain between any two nodes $i$ and $j, i \neq j, \sigma_{j}^{2}$ is the noise power at node $j$, and $g_{j j}$ is the selfinterference channel gain at node $j$. Note the RSI term in the denominator of the SNR. According to our earlier assumption, $\chi_{a}=\chi_{b} \triangleq \chi_{1}\left(\right.$ also, $\left.\chi_{c}=\chi_{d} \triangleq \chi_{2}\right)$.

Under (HD, FD) (i.e., $P_{1}$ selects HD while $P_{2}$ selects FD), $P_{1}^{\prime}$ 's utility, denoted by $U_{1}^{(\mathrm{HF})}$, includes the throughput of the forward link minus its transmission cost. The same amount of external interference is still induced on $P_{1}$, as $P_{2}$ still operates in FD mode. Formally, $U_{1}^{(\mathrm{HF})}$ can be written as:

$$
\begin{equation*}
U_{1}^{(\mathrm{HF})}=\mathbb{1}_{a b}^{(\mathrm{HF})} R_{a b}^{(\mathrm{HF})}-C_{a}=\mathbb{1}_{a b}^{(\mathrm{HF})} \log \left(1+\gamma_{a b}^{(\mathrm{HF})}\right)-c_{a} P_{t, a} \tag{3}
\end{equation*}
$$

where $\gamma_{a b}^{(\mathrm{HF})}=P_{t, a} g_{a b} /\left(P_{t, c} g_{c b}+P_{t, d} g_{d b}+\sigma_{b}^{2}\right)$ is the SNR at $b$ when receiving data from $a$ under (HD, FD).

For the two other strategy profiles, $P_{1}$ 's utility can be written as follows:

$$
\begin{align*}
U_{1}^{(\mathrm{FH})}= & \mathbb{1}_{a b}^{(\mathrm{FH})} \log \left(1+\gamma_{a b}^{(\mathrm{FH})}\right)-c_{a} P_{t, a} \\
& +\mathbb{1}_{b a}^{(\mathrm{FH})} \log \left(1+\gamma_{b a}^{(\mathrm{FH})}\right)-c_{b} P_{t, b}  \tag{4}\\
U_{1}^{(\mathrm{HH})}= & \mathbb{1}_{a b}^{(\mathrm{HH})} \log \left(1+\gamma_{a b}^{(\mathrm{HH})}\right)-c_{a} P_{t, a} \tag{5}
\end{align*}
$$

where $\gamma_{a b}^{(\mathrm{FH})}=P_{t, a} g_{a b} /\left(\chi_{b}^{2} P_{t, b} g_{b b}+P_{t, c} g_{c b}+\sigma_{b}^{2}\right), \gamma_{b a}^{(\mathrm{FH})}=P_{t, b} g_{b a} /$ $\left(\chi_{a}^{2} P_{t, a} g_{a a}+P_{t, c} g_{c a}+\sigma_{a}^{2}\right)$, and $\gamma_{a b}^{(\mathrm{HH})}=P_{t, a} g_{a b} /\left(P_{t, c} g_{c b}+\sigma_{b}^{2}\right)$.

To find the BNE, we first define technical conditions (TCs) under which no outage/collision occurs at the receivers of both players. We start by analyzing the no-outage case to facilitate understanding the Bayesian game analysis. Then, we conclude this section by deriving the BNE for the general case when outage is possible.

Outage will not occur for $P_{1}$ iff $\gamma_{a b}^{m n} \geq \gamma_{a b}^{*}$ and $\gamma_{b a}^{m n} \geq \gamma_{b a}^{*}$, $\forall m, n \in S_{1}$. However, by examining the SNR expressions for different strategy profiles, we notice that the minimum SNR is encountered under (FD, FD) due to the existence of two interference terms from nodes $c$ and $d$, in addition to the RSI. With the imposed conditions on $\gamma_{a b}^{(\mathrm{FF})}$ and $\gamma_{b a}^{(\mathrm{FF})}$, we can implicitly ensure that no outage occurs under the other strategy profiles. Formally, we define the following two TCs $P_{1}$ :

$$
\begin{gather*}
\mathrm{TC}_{11}: P_{t, c} g_{c b}+P_{t, d} g_{d b}-\frac{P_{t, a} g_{a b}}{\gamma_{a b}^{*}}+\chi_{1}^{2} P_{t, b} g_{b b}+\sigma_{b}^{2}<0  \tag{6}\\
\mathrm{TC}_{12}: P_{t, c} g_{c a}+P_{t, d} g_{d a}-\frac{P_{t, b} g_{b a}}{\gamma_{b a}^{*}}+\chi_{1}^{2} P_{t, a} g_{a a}+\sigma_{a}^{2}<0 \tag{7}
\end{gather*}
$$

Two similar TCs can be established for $P_{2}$. We later discuss the case when those technical conditions are not satisfied (i.e., an outage may occur).

When $\chi_{i}$ is sufficiently small, $P_{i}$ can efficiently suppress its self-interference signal, ensuring that the FD strategy
dominates the HD strategy. $\chi_{1}^{*}$ is the value of $\chi_{1}$ at which $U_{1}^{(\mathrm{FH})}=U_{1}^{(\mathrm{HH})}$ and $\chi_{2}^{*}$ is the value of $\chi_{2}$ at which $U_{2}^{(\mathrm{HF})}=$ $U_{2}^{(\mathrm{HH})}$. As $\chi_{i}$ increases, the higher RSI makes the HD strategy more preferable in some cases (especially if the other player is following the HD strategy). This means that $P_{1}$ 's utility, for example, under (FD, HD) decreases faster with $\chi_{1}$ than that under (FD, FD). The reason for this is that under (FD, HD), the RSI dominates the external interference (since $P_{2}$ is playing HD), which is not the case for the (FD, FD). The same argument applies to player $P_{2} \cdot \chi_{1}^{* *}$ is defined as the value of $\chi_{1}$ at which $U_{1}^{(\mathrm{FF})}=U_{1}^{(\mathrm{HF})}$ and $\chi_{2}^{* *}$ is defined as the value of $\chi_{2}$ at which $U_{2}^{(\mathrm{FF})}=U_{2}^{(\mathrm{FH})}$. As $\chi_{i}$ continues to increase beyond $\chi_{i}^{* *}$, the RSI signal will be very high, forcing the utilities under the FD strategy to be lower than that of the HD strategy (irrespective of the other player's strategy). Towards the end of this section, we discuss how $\chi_{i}^{*}$ and $\chi_{i}^{* *}$ can be computed.

### 4.2 Existence and Uniqueness of a Bayesian NE

Theorem 1. Game $\mathcal{G}$ has a $\operatorname{BNE}\left(s_{1}^{*}, s_{2}^{*}\right)$, which is given by:

$$
s_{i}^{*}=\left\{\begin{array}{lll}
\text { FD, } & \text { if } \theta_{i}=1  \tag{8}\\
\begin{cases}\text { FD, } & \text { if } p_{\hat{i} 1}>\alpha_{i} \\
\text { FD or HD, } & \text { if } p_{\hat{i} 1} \leq \alpha_{i} \text { and } \\
\text { HD, } & p_{\hat{i 1} 1}+p_{\hat{i} 2} \geq \alpha_{i} \\
\text { if } \theta_{i}=2 \\
\text { HD, } & \text { if } p_{\hat{i} 1}+p_{\hat{i} 2}<\alpha_{i} \\
\text { if } \theta_{i}=3,\end{cases}
\end{array}\right.
$$

for $i \in\{1,2\} . \hat{i}$ is the index of node $i$ 's peer node (i.e., if $i=1, \hat{i}=2$, and vice verse). $\alpha_{1}, \alpha_{2} \in[0,1]$ are defined as:

$$
\begin{align*}
& \alpha_{1} \stackrel{\text { def }}{=} \frac{U_{1}^{(\mathrm{HH})}-U_{1}^{(\mathrm{FH})}}{U_{1}^{(\mathrm{HH})}-U_{1}^{(\mathrm{FH})}+U_{1}^{(\mathrm{FF})}-U_{1}^{(\mathrm{HF})}}  \tag{9}\\
& \alpha_{2} \stackrel{\text { def }}{=} \frac{U_{2}^{(\mathrm{HH})}-U_{2}^{(\mathrm{HF})}}{U_{2}^{(\mathrm{HH})}-U_{2}^{(\mathrm{HF})}+U_{2}^{(\mathrm{FF})}-U_{2}^{(\mathrm{FH})}} \tag{10}
\end{align*}
$$

Note that $\alpha_{i}$ is a function of $\chi_{i}, i \in\{1,2\}$.
Proof. If player $P_{1}$ is of type 1 , then $U_{1}^{(\mathrm{FF})}>U_{1}^{(\mathrm{HF})}$ and $U_{1}^{(\mathrm{FH})}>$ $U_{1}^{(\mathrm{HH})}$, i.e., the FD strategy strictly dominates the HD strategy. On the other hand, if $P_{1}$ is of type 3 , then $U_{1}^{(\mathrm{FF})}<$ $U_{1}^{(\mathrm{HF})}$ and $U_{1}^{(\mathrm{FH})}<U_{1}^{(\mathrm{HH})}$, i.e., the HD strategy strictly dominates the FD strategy. Applying the same argument to player $P_{2}$, we can determine the BNE if the players are of type 1 or 3 , as shown in (8).

The more challenging case occurs when at least one of the players is of type 2. If $P_{1}$ is of type 2 , then $U_{1}^{(\mathrm{FF})}>$ $U_{1}^{(\mathrm{HF})}$ and $U_{1}^{(\mathrm{FH})}<U_{1}^{(\mathrm{HH})}$. Therefore, $P_{1}^{\prime}$ 's decision depends on $P_{2}{ }^{\prime}$ s action. As mentioned earlier, $P_{2}{ }^{\prime}$ s action will be FD if $\theta_{2}=1$ and HD if $\theta_{2}=3$, however, its action when $\theta_{2}=2$ is not determined yet. First, consider the case when $P_{2}$ 's action is FD and $\theta_{2}=2$. In this case, $P_{1}$ 's FD strategy dominates the HD strategy iff: $p_{21} U_{1}^{(\mathrm{FF})}+p_{22} U_{1}^{(\mathrm{FF})}+p_{23} U_{1}^{(\mathrm{FH})}>p_{21} U_{1}^{(\mathrm{HF})}+p_{22} U_{1}^{(\mathrm{HF})}+p_{23} U_{1}^{(\mathrm{HH})}$.

Second, if $P_{2}{ }^{\prime}$ s action is HD when $\theta_{2}=2$, then $P_{1}{ }^{\prime}$ s FD strategy dominates the HD strategy iff:

$$
p_{21} U_{1}^{(\mathrm{FF})}+p_{22} U_{1}^{(\mathrm{FH})}+p_{23} U_{1}^{(\mathrm{FH})}>p_{21} U_{1}^{(\mathrm{HF})}+p_{22} U_{1}^{(\mathrm{HH})}+p_{23} U_{1}^{(\mathrm{HH})}
$$

Solving the above two equations along with $p_{21}+$ $p_{22}+p_{23}=1$, we obtain the value of $\alpha_{1}$ and derive the conditions on $P_{2}$ 's probability distribution, as shown in (8). A similar approach is used to determine $\alpha_{2}$ for $P_{2}$.

The derived BNE, which represents the stable point for both players, can be interpreted as follows. Since $P_{i}$ knows its own true type and the probability distribution over the types of the other player, the first step for player $P_{i}$ is to check its own type. This can be done by calculating its SIC capability, $\chi_{i}$, by comparing the power of the residual selfinterference signal to its original transmission power. If $\chi_{i} \leq \chi_{i}^{*}$, then $P_{i}$ is of type 1. In this case, $P_{i}$ plays FD irrespective of what the other player is playing. If $\chi_{i} \geq \chi_{i}^{* *}$, then $P_{i}$ is of type 3. In this case, $P_{i}$ plays HD irrespective of what the other player is playing. Otherwise (i.e., $\chi_{i}^{*}<\chi_{i}<\chi_{i}^{* *}$ ), player $P_{i}$ is of type 2 . In this case, $P_{i}$ has to use the probability distribution over the other player types to determine its action by following Theorem 1.

Corollary 1. If each of the two players $P_{1}$ and $P_{2}$ satisfies one of the following two conditions:

$$
\begin{gather*}
p_{\hat{i} 1}>\alpha_{i}  \tag{11}\\
p_{\hat{i} 1}+p_{\hat{i} 2}<\alpha_{i} \tag{12}
\end{gather*}
$$

where $i \in\{1,2\}$ and $\hat{i}$ is the index of node $i$ 's peer node, then game $\mathcal{G}$ has a unique BNE, which is given by:

$$
\begin{gather*}
((\mathrm{F}, \mathrm{~F}, \mathrm{H}),(\mathrm{F}, \mathrm{~F}, \mathrm{H})) \text { if } p_{21}>\alpha_{1} \& p_{11}>\alpha_{2}  \tag{13}\\
((\mathrm{~F}, \mathrm{~F}, \mathrm{H}),(\mathrm{F}, \mathrm{H}, \mathrm{H})) \text { if } p_{21}>\alpha_{1} \& p_{11}+p_{12}<\alpha_{2}  \tag{14}\\
((\mathrm{~F}, \mathrm{H}, \mathrm{H}),(\mathrm{F}, \mathrm{~F}, \mathrm{H})) \text { if } p_{21}+p_{22}<\alpha_{1} \& p_{11}>\alpha_{2}  \tag{15}\\
((\mathrm{~F}, \mathrm{H}, \mathrm{H}),(\mathrm{F}, \mathrm{H}, \mathrm{H})) \text { if } p_{21}+p_{22}<\alpha_{1} \& p_{11}+p_{12}<\alpha_{2} \tag{16}
\end{gather*}
$$

where the notation $((\mathrm{F}, \mathrm{F}, \mathrm{H}),(\mathrm{F}, \mathrm{H}, \mathrm{H}))$, for instance, means that $P_{1}$ operates in the FD mode when $\theta_{1}=1$ or 2 and in the HD mode when $\theta_{1}=3$, while $P_{2}$ operates in the FD mode when $\theta_{2}=1$ and in the HD mode when $\theta_{2}=2$ or 3 .

Proof. The two-player Bayesian game $\mathcal{G}$ (with each player having one of three possible types) can be equivalently formulated as a $2^{3}$-by- $2^{3}$ strategic game. From Theorem 1 and its proof, it can be shown that under each of the conditions stated in (13), (14), (15), and (16), the corresponding strategy profile consists of a strictly dominating strategy for each player in the 8-by-8 game. Hence, the BNE is unique.
Corollary 2. If exactly one of the players satisfies neither (11) nor (12), then game $\mathcal{G}$ has two BNEs (from the strategy profiles given by (13), (14), (15), and (16)).
Proof. Based on Theorem 1, player $P_{i}$ always plays FD if $\theta_{i}=1$ and always plays HD if $\theta_{i}=3$. If $\theta_{i}=2$, then the BNE is decided based on the relation between $p_{\hat{i} 1}, p_{\hat{i} 2}$, and $\alpha_{i}$. Specifically, if (11) or (12) is satisfied, then player $P_{i}$ plays FD or HD, respectively. However, if neither conditions are satisfied, then playing FD or HD is equivalent (see proof of Theorem 1).

While Corollary 1 illustrates the case where each of the two players $P_{1}$ and $P_{2}$ satisfies one of the two conditions shown in (11) and (12), Corollary 2 focuses on the scenario where exactly one of the players neither satisfies (11) nor (12). Under this scenario, one player can either play HD or FD (both are equivalent), while the other player has to play only a single strategy (either HD or FD based on Theorem 1 rules). Hence, two of the four BNEs defined in Corollary 1 will be valid simultaneously under Corollary 2. As an example, if we assume that only $P_{1}$ satisfies neither (11) nor (12), then the two BNEs will be ((F, F, H), (F, F, H) ) and ((F, F, H), (F,H,H)) if $p_{21}>\alpha_{1}$. On the other hand, if $p_{21}+p_{22}<\alpha_{1}$, then the two BNEs will be ( $\mathrm{F}, \mathrm{H}$, H), (F, F, H)) and ((F,H,H), (F, H, H)).

Corollary 3. If both players satisfy neither (11) nor (12), then the game has the four BNEs, given by (13)-(16).
Proof. The proof of Corollary 3 is very similar to that of Corollary 2 (it follows directly from Theorem 1 and Corollary 1). When both players satisfy neither (11) nor (12), then both players can either play FD or HD. Hence, the game has the four BNEs, given by (13), (14), (15), (16).

### 4.3 Accounting for Link Outages

We now relax the technical conditions ((6) and (7) for $P_{1}$ and two similar conditions for $P_{2}$ ), and assume that any of the forward/reverse links may experience outage. In this case, the relation between $\chi_{i}^{*}$ and $\chi_{i}^{* *}$ is not fixed (i.e., $\chi_{i}^{*}$ could be larger or smaller than $\left.\chi_{i}^{* *}\right)$. The reason is that in the case of outage, if $P_{2}$ operates in the FD mode, then $U_{1}^{(\mathrm{FF})}$ may decrease faster with $\chi_{1}$ compared to $U_{1}^{(\mathrm{FH})}$, and hence as $\chi_{1}$ increases the condition $U_{1}^{(\mathrm{FF})}=U_{1}^{(\mathrm{HF})}$ may be satisfied before the condition $U_{1}^{(\mathrm{FH})}=U_{1}^{(\mathrm{HH})}$. Therefore, $\chi_{1}^{* *}<\chi_{1}^{*}$ (recall the definition of $\chi_{1}^{*}$ and $\chi_{1}^{* *}$ in Section 4.1). The same situation arises in the case of $P_{2}$ under the possibility of outage.
Theorem 2. Theorem 1 and Corollaries 1-3 hold true for game $\mathcal{G}$ under the possibility of outage. However, $\theta_{i}, i \in\{1,2\}$, in this case, is given by:

$$
\theta_{i}= \begin{cases}1, & \text { if } \chi_{i} \leq \min \left(\chi_{i}^{*}, \chi_{i}^{* *}\right)  \tag{17}\\ 2, & \text { if } \min \left(\chi_{i}^{*}, \chi_{i}^{* *}\right)<\chi_{i}<\max \left(\chi_{i}^{*}, \chi_{i}^{* *}\right) \\ 3, & \text { if } \chi_{i} \geq \max \left(\chi_{i}^{*}, \chi_{i}^{* *}\right)\end{cases}
$$

Proof. From the definitions of $\chi_{i}^{*}$ and $\chi_{i}^{* *}$ in Section 4.1, it can be shown that irrespective whether outage occurs or not, the dominating strategy of player $P_{i}$ when $\chi_{i} \leq \min \left(\chi_{i}^{*}, \chi_{i}^{* *}\right)$ is FD, and when $\chi_{i} \geq \max \left(\chi_{i}^{*}, \chi_{i}^{* *}\right)$ is HD. When $\min \left(\chi_{i}^{*}, \chi_{i}^{* *}\right)<\chi_{i}<\max \left(\chi_{i}^{*}, \chi_{i}^{* *}\right)$, following the same approach in the proof of Theorem 1 (by deriving the conditions under which $P_{i}$ 's expected utility under the FD strategy is higher than that under the HD strategy while fixing the other player strategy, and solving the two resulting inequalities along with $p_{21}+p_{22}+p_{23}=1$ ), it can be shown that the dominating strategy is given by (8) when $\theta_{i}=2$. Hence, Theorem 2 holds.

Next, we derive closed-form expressions for $\chi_{1}^{*}$ and $\chi_{1}^{* *}$. In this derivation, we focus on player $P_{1}$; a similar approach could be used to derive the thresholds of player $P_{2}$. The challenge in this derivation is the presence of the indicator functions along with the logarithmic terms.

We start by deriving $\chi_{1}^{* *}$, which occurs when $U_{1}^{(\mathrm{FF})}=$ $U_{1}^{(\mathrm{HF})}$. Substituting in the previous equation using (1) and (3), we get the following:

$$
\begin{equation*}
\mathbb{1}_{a b}^{(\mathrm{FF})} R_{a b}^{(\mathrm{FF})}+\mathbb{1}_{b a}^{(\mathrm{FF})} R_{b a}^{(\mathrm{FF})}-C_{a}-C_{b}=\mathbb{1}_{a b}^{(\mathrm{HF})} R_{a b}^{(\mathrm{HF})}-C_{a} \tag{18}
\end{equation*}
$$

Rearranging the above equation, we get $\mathbb{1}_{a b}^{(\mathrm{FF})} R_{a b}^{(\mathrm{FF})}+\mathbb{1}_{b a}^{(\mathrm{FF})}$ $R_{b a}^{(\mathrm{FF})}=w$, where $w \stackrel{\text { def }}{=} \mathbb{1}_{a b}^{(\mathrm{HF})} R_{a b}^{(\mathrm{HF})}+C_{b}$ is a term that does not depend on $\chi_{1}$.

In the following analysis, we assume that the link throughput is measured in nats per second (i.e., using the natural logarithm). Substituting the expressions for $R_{a b}^{(\mathrm{FF})}$ and $R_{b a}^{(\mathrm{FF})}$ in (2), we get the following:

$$
\begin{align*}
w= & \mathbb{1}_{a b}^{(\mathrm{FF})} R_{a b}^{(\mathrm{FF})}+\mathbb{1}_{b a}^{(\mathrm{FF})} R_{b a}^{(\mathrm{FF})} \\
= & \mathbb{1}_{a b}^{(\mathrm{FF})} \ln \left(1+\frac{P_{t, a} g_{a b}}{\chi_{1}^{* * 2} P_{t, b} g_{b b}+P_{t, c} g_{c b}+P_{t, d} g_{d b}+\sigma_{b}^{2}}\right) \\
& +\mathbb{1}_{b a}^{(\mathrm{FF})} \ln \left(1+\frac{P_{t, b} g_{b a}}{\chi_{1}^{* * 2} P_{t, a} g_{a a}+P_{t, c} g_{c a}+P_{t, d} g_{d a}+\sigma_{a}^{2}}\right) \\
= & \ln \left(1+\frac{P_{t, a} g_{a b}}{\chi_{1}^{* * 2} P_{t, b} g_{b b}+P_{t, c} g_{c b}+P_{t, d} g_{d b}+\sigma_{b}^{2}}\right)^{\mathbb{1}_{a b}^{(\mathrm{FF})}} \\
& +\ln \left(1+\frac{P_{t, b} g_{b a}}{\chi_{1}^{* * 2} P_{t, a} g_{a a}+P_{t, c} g_{c a}+P_{t, d} g_{d a}+\sigma_{a}^{2}}\right)^{\mathbb{1}_{b a}^{(\mathrm{FF})}} \\
w= & \ln \left[\left(1+\frac{P_{t, a} g_{a b}}{\chi_{1}^{* * 2} P_{t, b} g_{b b}+u}\right)^{\left.\mathbb{1}_{a b}^{(\mathrm{FF})}\left(1+\frac{P_{t, b} g_{b a}}{\chi_{1}^{* * 2} P_{t, a} g_{a a}+v}\right)^{\mathbb{1}_{b a}^{(\mathrm{FF})}}\right]}\right. \tag{19}
\end{align*}
$$

where $u \stackrel{\text { def }}{=} P_{t, c} g_{c b}+P_{t, d} g_{d b}+\sigma_{b}^{2}$ and $v \stackrel{\text { def }}{=} P_{t, c} g_{c a}+P_{t, d} g_{d a}+\sigma_{a}^{2}$ are independent of $\chi_{1}$.

To obtain an expression for $\chi_{1}^{* *}$, we need to consider different combinations of the values of the indicator functions $\mathbb{1}_{a b}^{(\mathrm{FF})}$ and $\mathbb{1}_{b a}^{(\mathrm{FF})}$ (four cases). For the first case $\left(\mathbb{1}_{a b}^{(\mathrm{FF})}=\right.$ $\mathbb{1}_{b a}^{(\mathrm{FF})}=0$ ), an outage will occur at both receivers of player $P_{1}$. In this case, $P_{1}$ 's utility under the FD strategy cannot be higher than his utility under the HD strategy. Hence, $\chi_{11}^{* *}=0$, where $\chi_{i j}^{* *}$ is defined as the value of $\chi_{i}^{* *}$ under case $j$.

The second case occurs when $\mathbb{1}_{a b}^{(\mathrm{FF})}=0$ and $\mathbb{1}_{b a}^{(\mathrm{FF})}=1$. Substituting with those values in (19) and after some manipulations, we get the following expression:

$$
\begin{equation*}
P_{t, b} g_{b a}=\left(e^{w}-1\right)\left(\chi_{12}^{* * 2} P_{t, a} g_{a a}+v\right) . \tag{20}
\end{equation*}
$$

Therefore, under case $2, \chi_{12}^{* *}$ can be expressed as follows:

$$
\begin{equation*}
\chi_{12}^{* *}=\sqrt{\left(\frac{1}{P_{t, a} g_{a a}}\right)\left(\frac{P_{t, b} g_{b a}}{e^{w}-1}-v\right)} \tag{21}
\end{equation*}
$$

Similarly, for case 3 , where $\mathbb{1}_{a b}^{(\mathrm{FF})}=1$ and $\mathbb{1}_{b a}^{(\mathrm{FF})}=0, \chi_{13}^{* *}$ can be expressed as follows:

$$
\begin{equation*}
\chi_{13}^{* *}=\sqrt{\left(\frac{1}{P_{t, b} g_{b b}}\right)\left(\frac{P_{t, a} g_{a b}}{e^{w}-1}-u\right)} \tag{22}
\end{equation*}
$$

Finally, under case 4 (i.e., $\mathbb{1}_{a b}^{(\mathrm{FF})}=1$ and $\mathbb{1}_{b a}^{(\mathrm{FF})}=1$ ), we substitute the values of the indicator functions in (19) and get the following equation:

$$
\begin{equation*}
\beta_{1} \chi_{14}^{* * 4}+\beta_{2} \chi_{14}^{* * 2}+\beta_{3}=0 \tag{23}
\end{equation*}
$$

where $\beta_{1}=P_{t, a} P_{t, b} g_{a a} g_{b b}\left(1-e^{w}\right)$, $\beta_{2}=\left(P_{t, b} g_{b b} v+P_{t, a} g_{a a} u\right)(1-$ $\left.e^{w}\right)+P_{t, b}^{2} g_{b a} g_{b b}+P_{t, a}^{2} g_{a b} g_{a a}$, and $\beta_{3}=u v\left(1-e^{w}\right)+P_{t, b} g_{b a} u+$ $P_{t, a} g_{a b} v+P_{t, a} P_{t, b} g_{a b} g_{b a}$. Equation (23) can be solved easily using the quadratic formula. To sum up the four cases, the value of $\chi_{1}^{* *}$ can be expressed as follows:

$$
\chi_{1}^{* *}= \begin{cases}0, & \text { if } \mathbb{1}_{a b}^{(\mathrm{FF})}=\mathbb{1}_{b a}^{(\mathrm{FF})}=0  \tag{24}\\ \sqrt{\left(\frac{1}{P_{t, a} g_{a a}}\right)\left(\frac{P_{t, b} g_{b a}}{e^{w}-1}-v\right),} & \text { if } \mathbb{1}_{a b}^{(\mathrm{FF})}=0, \mathbb{1}_{b a}^{(\mathrm{FF})}=1 \\ \sqrt{\left(\frac{1}{P_{t, b} g_{b b}}\right)\left(\frac{P_{t, a} g_{a b}}{e^{W}-1}-u\right),} & \text { if } \mathbb{1}_{a b}^{(\mathrm{FF})}=1, \mathbb{1}_{b a}^{(\mathrm{FF})}=0 \\ \sqrt{\frac{-\beta_{2}-\sqrt{\beta_{2}^{2}-4 \beta_{1} \beta_{3}}}{2 \beta_{1}},} & \text { if } \mathbb{1}_{a b}^{(\mathrm{FF})}=\mathbb{1}_{b a}^{(\mathrm{FF})}=1\end{cases}
$$

The above approach can also be used to derive $\chi_{1}^{*}$, resulting in:

$$
\chi_{1}^{*}= \begin{cases}0, & \text { if } \mathbb{1}_{a b}^{(\mathrm{FH})}=\mathbb{1}_{b a}^{(\mathrm{FH})}=0  \tag{25}\\ \sqrt{\left(\frac{1}{P_{t, a} g_{a a}}\right)\left(\frac{P_{t, b} g_{b a}}{e^{\hat{\omega}}-1}-\hat{v}\right),} & \text { if } \mathbb{1}_{a b}^{(\mathrm{FH})}=0, \mathbb{1}_{b a}^{(\mathrm{FH})}=1 \\ \sqrt{\left(\frac{1}{P_{t, b} g_{b b}}\right)\left(\frac{P_{t, a} g_{a b}}{e^{\hat{\omega}}-1}-\hat{u}\right),} & \text { if } \mathbb{1}_{a b}^{(\mathrm{FH})}=1, \mathbb{1}_{b a}^{(\mathrm{FH})}=0 \\ \sqrt{\frac{-\hat{\hat{\beta}_{2}}-\sqrt{\hat{\beta}_{2}^{2}-4 \hat{\beta}_{1} \hat{\beta}_{3}}}{2 \hat{\beta}_{1}},} & \text { if } \mathbb{1}_{a b}^{(\mathrm{FH})}=\mathbb{1}_{b a}^{(\mathrm{FH})}=1\end{cases}
$$

where $\hat{w} \stackrel{\text { def }}{=} \mathbb{1}_{a b}^{(\mathrm{HH})} R_{a b}^{(\mathrm{HH})}+C_{b}, \hat{u} \stackrel{\text { def }}{=} P_{t, c} g_{c b}+\sigma_{b}^{2}$ and $\hat{v} \stackrel{\text { def }}{=} P_{t, c} g_{c a}+$ $\sigma_{a}^{2}, \hat{\beta_{1}}=P_{t, a} P_{t, b} g_{a a} g_{b b}\left(1-e^{\hat{w}}\right), \hat{\beta}_{2}=\left(P_{t, b} g_{b b} \hat{v}+P_{t, a} g_{a a} \hat{u}\right)\left(1-e^{\hat{\omega}}\right)+$ $P_{t, b}^{2} g_{b a} g_{b b}+P_{t, a}^{2} g_{a b} g_{a a}$, and $\hat{\beta_{3}}=\hat{u} \hat{v}\left(1-e^{\hat{w}}\right)+P_{t, b} g_{b a} \hat{u}+P_{t, a} g_{a b} \hat{v}+$ $P_{t, a} P_{t, b} g_{a b} g_{b a}$.

## 5 Extension to Multiple Links with Heterogeneous SiC Capabilities

In Section 4, we studied the coexistence problem between two links with heterogeneous SIC capabilities in a Bayesian game-theoretic framework. In this section, we extend the treatment to more than two links.

### 5.1 Approximation of the Network Interference Graph

Due to the complexity of deriving the BNE of a game that involves more than two links, we simplify the interactions (i.e., interference relations) between the multiple FD-enabled links. Specifically, we approximate the network interference graph, as depicted in Fig. 2. In the exact interference graph, shown in Fig. 2a, each link is affected by all other links in the network. Instead, in the proposed approximation, we assume that each link is only affected by a single link in the network, which is its dominant interfering link. ${ }^{1}$ We consider a distance-dependent interference (i.e., path-loss), where the dominant interfering link of, say link 1, will be the closest link. Because links are FD-enabled, both nodes of a link are transceivers, and we compute the distance between two links as the distance between the midpoints of the segments that connect the two transceivers of each link.

1. This assumption has been used in the wireless communications and networking literature [36], [37].


Fig. 2. Approximation of the network interference graph.

### 5.2 FD/HD Mode Selection

In this section, we first derive the structure of the network interference graph under the aforementioned approximation. Then, we discuss our FD/HD mode selection game.
Proposition 1. The approximate network interference graph consists of loops, each comprised of two links only, with branches attached to them, as shown in Fig. 3. Note that an interference graph, in general, have multiple loops. The figure shows only one of them.

Proof. To derive the structure of the approximate interference graph, let us start with one FD-enabled link, say link 1 in Fig. 4a, and its dominant interfering link, say link 2. Then, the dominant interfering link of link 2 can be one of the following:

- Link 1 , in which case, links 1 and 2 will be engaged in a two-player game, or
- Another link, say link 3 in Fig. 4a, in which case, the distance between links 2 and 3, denoted by $d_{23}$, needs to be smaller than the distance between links 1 and $2\left(d_{12}\right)$. At the same time, $d_{13}$ needs to be larger than $d_{12}$ (because link 2 is the dominant interfering link of link 1). Hence, $d_{13}>d_{12}>d_{23}$.
Now, if we assume that there are only three links in the network, then link 2 is also the dominant interfering link of link 3, and links 2 and 3 will be engaged in a two-player game. The interference graph in this case is shown in Fig. 4a. On the other hand, if we assume that there is another link, say link 4 , that is closer to link 3 than link 2 (i.e., $d_{34}<d_{23}$ ), as shown in Fig. 4b, then, $d_{34}<d_{23}<d_{24}$, where the last inequality is because link 3 is the dominant interfering link of link 2. Furthermore, $d_{34}<d_{23}<d_{12}<$ $d_{14}$, where the last two inequalities are because link 3 is the dominant interfering link of link 2 and link 2 is the dominant interfering link of link 1 , respectively. Since $d_{34}$ is smaller than both $d_{24}$ and $d_{14}$, link 3 is also the dominant interfering link of link 4 , and links 3 and 4 will be engaged in a two-player game. The interference graph in this case is shown in Fig. 4b.

From the above discussion, we can see that the approximate interference graph consists of loops, each of


Fig. 3. Generalized approximate network interference graph.


Fig. 4. Approximate network interference graph for three and four FDenabled links.
two links only, with branches attached to them, as shown in Fig. 3.
Based on the above, the unique BNE of the multi-player game can be obtained by applying Algorithm 1.

## Algorithm 1. Finding the Unique BNE of the MultiPlayer Game

Identify all loops in the network.
Solve their corresponding two-player Bayesian games (i.e., compute the BNE), as in Section 4.
for each loop do
if there is a link attached to this loop (e.g., links 3 and 4 in
Fig. 3) then
Compute its best-response strategy to the derived
BNE in Step (1).
end if
if there is a link that is two-hop away from the loop (e.g., link 5 in Fig. 3) then
Compute its best-response strategy to the best- response strategy computed in the previous step.
end if
Continue in the same way until reaching the leafs of the interference graph.
end for

## 6 Performance Evaluation

In this section, we conduct numerical evaluations and LabVIEW/MATLAB simulations to assess the performance of our proposed game-theoretic analysis.

### 6.1 Numerical Results

In this section, we evaluate via numerical analysis the characteristics and structure of the BNE for the two-player game. Unless stated otherwise, we use the following parameter values: The channel gain between nodes $i$ and $j, g_{i j}$, is set to $1, \forall i, j \in\{a, b, c, d\}, \sigma_{i}^{2}=1$ Watt, and the normalized transmission $\operatorname{cost} c_{i}$ is set 0.001 .

Fig. 5 depicts the expected utility for $P_{1}$ versus $\chi_{1}$ at different values of interference channel gains (ICGs) (i.e., channel gains between the transmitters of $P_{1}$ and the receivers of $P_{2}$, and vice versa). As shown in the figure, increasing the ICGs reduces the expected utility of $P_{1}$. If $P_{1}$ is of type 1 , its expected utility decreases with $\chi_{1}$, as $P_{1}$ operates in the FD mode, whereas when it is of type 3 , its expected utility is constant because it operates in the HD mode. When $P_{1}$ is of type 2, $P_{1}$ starts operating in the FD mode as $\chi_{1}$ increases (since $p_{21}>\alpha_{1}$ ), and eventually it switches to the HD mode when $\alpha_{1}$ exceeds $p_{21}$. The expected utility


Fig. 5. $P_{1}$ 's expected utility at BNE versus $\chi_{1}\left(\chi_{2}=0\right.$, uniform distribution over the types of both players, and $P_{t, i}=100$ Watts $\forall i$ ).


Fig. 6. $P_{1}$ 's expected utility at BNE versus $\chi_{1}\left(\chi_{2}=0\right.$, uniform distribution over $P_{1}$ types, $p_{22}=p_{23}=\left(1-p_{21}\right) / 2$, ICGs $=0.05$, and $P_{t, i}=100$ Watts $\forall i)$.


Fig. 7. SIC thresholds for $P_{1}$ and $P_{2}$ versus $P_{t, c}$ (ICGs $=0.05$ and $P_{t, i}=100$ Watts $\left.\forall i\right)$.


Fig. 8. Expected utilities of $P_{1}$ and $P_{2}$ at BNE versus $p_{21}\left(\chi_{1}=\chi_{2}=0\right.$, ICGs $=0.05$, and maximum transmission powers).
when $\theta_{1}=2$ first decreases with $\chi_{1}$ and then remains constant. Note that the SIC thresholds are different for different ICGs (we only show these thresholds when ICG $=0.01$ ). In Fig. 6, we show $P_{1}$ 's expected utility for different values of $p_{21}$ (a priori probability that $P_{2}$ is of type 1). As $p_{21}$ increases, $P_{1}$ 's expected utility decreases. The reason is that if $p_{21}$ is sufficiently small (e.g., $p_{21}=0.1$ ), the expected interference level is relatively low and hence higher expected utility can be achieved.

Fig. 7 shows the SIC thresholds of $P_{1}$ and $P_{2}$ versus $P_{t, c}$. Under conditions (6) and (7) for $P_{1}$ and the two similar conditions for $P_{2}, \chi_{i}^{*}<\chi_{i}^{* *} \forall i$. Because $P_{t, c}$ is the dominating term in $\gamma_{a b}^{(\mathrm{HH})}$, increasing $P_{t, c}$ causes $U_{1}^{(\mathrm{HH})}$ to decrease faster


Fig. 9. Player $P_{1}$ 's utility versus its transmission powers, $P_{t, a}$ and $P_{t, b}$ ( $\chi_{1}=0, P_{2}$ operates in the HD mode, and $P_{t, c}=100$ Watts).


Fig. 10. Player $P_{1}$ 's utility versus its transmission powers, $P_{t, a}$ and $P_{t, b}$ ( $\chi_{1}=0.4, P_{2}$ operates in the HD mode, and $P_{t, c}=100$ Watts).


Fig. 11. $P_{1}$ 's expected utility at BNE versus $c\left(\chi_{2}=0\right.$, uniform distribution over the types of both players, and $P_{t, i}=100$ Watts $\forall i$ ).
than $U_{1}^{(\mathrm{FH})}$. Hence, $\chi_{1}^{*}$ increases with $P_{t, c}$. A similar argument can be made regarding $\chi_{1}^{* *}$. On the other hand, increasing $P_{t, c}$ causes $U_{2}^{(\mathrm{HF})}$ to decrease faster than $U_{2}^{(\mathrm{HH})}$. Hence, $\chi_{2}^{*}$ decreases with $P_{t, c}$. In Fig. 8, we plot the players' expected utilities versus $p_{21}$ for different values of $p_{11}$. As $p_{21}$ increases, $P_{1}$ 's expected utility decreases. Similarly, $P_{2}$ 's expected utility decreases with $p_{11}$, whereas it is constant with $p_{21}$, because $P_{2}$ already knows its type.

In the next two figures, we consider the case where an outage may occur at the receiver of any player. Even with complete SIC, when both players operate in the FD mode, outage may occur at all receivers, resulting in negative utilities for both players (because of the transmission costs). However, when $P_{2}$ operates in the HD mode, $P_{1}$ starts getting a positive utility, as shown in Fig. 9. Note that $P_{1}$ doubles its throughput in the (FD, HD) strategy profile compared to (HD, HD) when the maximum transmission power is used. However, as $P_{1}^{\prime}$ 's transmission power decreases, outage occurs. Fig. 10 shows the case when $\chi_{1}=0.4$. In this case, $P_{1}$ 's utility under (HD, HD) with $P_{t, c}=10$ Watts is higher than that under (FD, HD). This is due to the increase in RSI level. Fig. 11 shows the variation of $P_{1}$ 's expected utility at BNE versus constant $c$, which is used to model the transmission power cost, at different SIC values and different ICGs. As $c$ increases, player 1 expected utility decreases linearly as more weight is given to the transmission power


Fig. 12. $P_{1}$ 's forward-link BER versus $\chi_{1}$ at different noise levels (no external interference, simulations).
cost. Increasing $\chi_{1}$ and the ICGs decreases $P_{1}$ 's expected utility due to the increase in the residual self-interference and the external interference, respectively.

### 6.2 Simulations

### 6.2.1 Two Links

In this section, we study via LabVIEW simulations the BER of different players assuming two links only. LabVIEW 2014 software, developed by National Instruments, is used to acquire, analyze, and visualize data. The simulation setup is as follows. We consider a single-channel FD network consisting of two links with different SIC capabilities. Each link can operate in the HD or FD modes. For transmission, we generate back-to-back packets, each consisting of 500 bits, modulated using QPSK and $1 / 2$ code rate. We then add control information (e.g., training sequence) and pass them to the pulse shaping block for transmission. The simulation parameters include the mean of the Gaussian noise (varied from -20 dBm to -50 dBm ), ICGs $=\{0.01,0.04,0.09\}$, and SIC capabilities (between 0 and 0.5). We focus on the more practical case where $\chi_{i}$ is low. Both the players' channel coefficient and the self-interference channel coefficient are fixed at $0.707+0.707 i$ (gain $=1$ ). For each simulation run, we take the average of 1000 iterations (i.e., 1000 back-to-back packets). We fix the channel phase at $\pi / 4$ and set the BER threshold needed for correct reception to $B E R_{\text {th }}=10^{-4}$.

Fig. 12 shows the BER for the forward-link of $P_{1}$ versus $\chi_{1}$ at different noise levels when $P_{2}$ is not transmitting. Depending on the noise level and the SIC capability, $P_{1}$ decides to operate in the HD or FD modes. This figure shows the effect of RSI (due to incomplete SIC) on the BER of $P_{1}$. As expected, the BER of $P_{1}$ increases with $\chi_{1}$. Specifically, $P_{1}$ operates in the FD mode for all values of $\chi_{1}$ at which the average $B E R$ is below $B E R_{\text {th }}$. Once the average BER exceeds


Fig. 13. Forward-link BER of $P_{1}$ versus $\chi_{1}$ at different noise levels. $P_{2}$ operates in the FD mode (ICGs $=0.01$, simulations).


Fig. 14. Forward-link BER of $P_{1}$ versus $\chi_{1}$ at different noise levels. $P_{2}$ operates in the HD mode (ICG $=0.01$, simulations).
$B E R_{\text {th }}$ (i.e., the receiver is not able to decode the packet), the optimal communication mode for $P_{1}$ becomes HD. As shown in Fig. 12, the threshold value of $\chi_{1}$ for switching from FD to HD decreases with the noise level; when the noise level is -50 dBm , this threshold equals 1 (i.e., $P_{1}$ always operates in the FD mode), whereas when the noise equals -20 dBm , this threshold equals 0.2 (i.e., $P_{1}$ operates in the FD mode if $\chi_{1}<0.2$ and operates in the HD mode otherwise).

Figs. 13 and 14 show the forward-link BER of $P_{1}$ versus $\chi_{1}$ at different noise levels, when $P_{2}$ operates in the FD and HD modes, respectively. For both figures, the channel coefficients are $0.0707+0.0707 i$ (ICGs $=0.01$ ). As expected, the threshold values of $\chi_{1}$ under the (FD, FD) strategy profile are lower than those under (FD, HD) for the same noise levels. This means that the range of SIC capabilities for $P_{1}$ to operate in the FD mode is narrower when $P_{2}$ operates in the FD mode compared to the case when $P_{2}$ operates in the HD mode. The reason is that when $P_{2}$ operates in the FD mode, it induces higher interference on $P_{1}$ than when it operates in the HD mode, which causes outage to occur sooner at $P_{1}$ (i.e., at smaller values of $\chi_{1}$ ). Comparing Fig. 12 to Figs. 13 and 14 , the range of $\chi_{1}$ under which $P_{1}$ operates in the FD mode is higher when $P_{2}$ is off compared to the case when it is transmitting. We repeat the above setup for different ICG values. We present samples of the results in Figs. 15 and 16. Fig. 15 shows the forward-link BER of $P_{1}$ versus $\chi_{1}$ at different noise levels, when $P_{2}$ operates in the FD mode. In this figure, the channel coefficients are $0.1414+0.1414 i$ (ICG $=0.04$ ). Fig. 16 shows the forward-link BER of $P_{1}$ versus $\chi_{1}$ at different noise levels, when $P_{2}$ operates in the HD mode and the ICG $=0.09$. Similar conclusions can be made regarding the regions of the FD and HD modes for $P_{1}$ as the ICGs increase and as $P_{2}$ changes its mode from HD to FD.


Fig. 15. Forward-link BER of $P_{1}$ versus $\chi_{1}$ at different noise levels. $P_{2}$ operates in the FD mode (ICGs $=0.04$, simulations).


Fig. 16. Forward-link BER of $P_{1}$ versus $\chi_{1}$ at different noise levels. $P_{2}$ operates in the HD mode (ICGs $=0.09$, simulations).

### 6.2.2 More Than Two Links

In this section, we evaluate the network utility under the BNE, considering a network with more than two FDenabled links in the network. The simulation setup is as follows. We consider a square area of $200 \times 200$ square meters, which contains multiple players (i.e., links) that are distributed according to a Poisson point process (PPP). Specifically, we randomly distribute the links' receivers in the specified area according to the PPP with parameter $\lambda$ $(\lambda \in[4,80])$. For each receiver, we assign a single transmitter whose location is uniformly distributed in a square of side length 20 meters around the corresponding receiver. We model the players' SIC capability $(\chi)$ as an uniformly distributed random variable in $\left[0, \chi_{\max }\right]\left(\chi_{\max }\right.$ is a controllable parameter). An example of this multi-link wireless network could be a residential area with multiple Wi-Fi APs that communicate with Wi-Fi stations (STAs) and coexist with multiple LTE-U small cells base stations in the 5 GHz band.

We model the wireless channels between nodes using the free-space path loss model. The transmission power is 20 dBm , receiver noise power is -90 dBm , antenna gain is 1 , carrier frequency is 5 GHz , and the path-loss exponent is 3.5. In the following simulation results and according to the discussion in Section 5, we first determine the loops in the network, where every two players belonging to a loop play the two-player Bayesian game discussed in Section 4 and take the corresponding action (i.e., play according to the derived BNE). In the second step, players that do not belong to any loop play their best-response strategy according to the received interference from their dominant link and the value of their own SIC capabilities. In our simulations, we compare players' sum utilities under the BNE with three basic schemes. 'All FD' and 'All HD' schemes correspond to 'all' players in the network playing FD and HD, respectively.


Fig. 17. Average network utility versus $\lambda$ (perfect SIC).


Fig. 18. Histogram of link actions under BNE versus $\lambda$ (perfect SIC).

The third scheme is called random in which all players select the FD action with probability 0.5 .

Fig. 17 shows average network utility versus $\lambda$ at perfect SIC for the four schemes under consideration (BNE, All FD, All HD, and random). Although all players have perfect SIC, playing FD all the time (i.e., 'All FD') is not the optimal scheme since external interference between players can be significant. Note that according to our earlier discussion in Section 4, the derived BNE is the equilibrium and optimal strategy profile. As expected, the sum utilities under the random scheme is between the 'All HD' and 'All FD' utilities. Fig. 18 shows the normalized histogram of various actions versus $\lambda$ under the BNE with perfect SIC. At low $\lambda$ values, the FD action dominates the HD action since the amount of external interference is small. As $\lambda$ increases (i.e., network becomes more dense), the FD action occurs less often, while the HD action occurs more often. Fig. 19 shows the average network utility versus $\lambda$ at $\chi_{\max }=2 \times 10^{-4}$ for the four schemes. In contrast to the perfect SIC case, in Fig. 19, the 'All HD' scheme outperforms the 'All FD' scheme, while the BNE returns the maximum utility for the whole network. Fig. 20 shows the normalized histogram of various actions versus $\lambda$ under the BNE at $\chi_{\max }=2 \times 10^{-4}$. Similar to Fig. 18, as $\lambda$ increases, the FD action occurs less often than the HD action. Note that the occurrence frequencies of the modes is varying very slowly with $\lambda$ since the HD mode is dominating the FD mode due to low SIC capabilities.

In the next set of simulations, we vary $\chi_{\text {max }}$ in the interval [ $0,3 \times 10^{-4}$ ] and compare the average network utility under the four schemes for different values of $\lambda$. Fig. 21 shows the average network utility versus $\chi_{\max }$ at $\lambda=20$. The average network utility under the BNE, 'All FD', and random schemes decreases with $\chi_{\max }$ due to the increase in the residual selfinterference signal. However, the average network utility is constant with $\chi_{\max }$ for the 'All HD' scheme. As shown in Fig. 21, the BNE returns the maximum sum utilities for the


Fig. 19. Average network utility versus $\lambda\left(\chi_{\max }=2 \times 10^{-4}\right)$.


Fig. 20. Histogram of link actions under BNE versus $\lambda\left(\chi_{\max }=2 \times 10^{-4}\right)$.


Fig. 21. Average network utility versus $\chi_{\max }(\lambda=20)$.


Fig. 22. Histogram of link actions under BNE versus $\chi_{\max }(\lambda=20)$.
whole network. Note also that at low $\chi_{\max }$, the 'All FD' strategy dominates the 'All HD' strategy since with good SIC capabilities, the amount of residual self-interference is very low. As $\chi_{\text {max }}$ increases, the average network utility under the 'All FD' strategy decreases, which makes the 'All HD' strategy dominates the 'All FD' and the random strategies. Furthermore, we found that under the BNE and at $\lambda=20$, the FD action occurs less frequently as $\chi_{\text {max }}$ increases, while that of the HD increases in occurrence as shown in Fig. 22.

Fig. 23 shows the average network utility versus $\chi_{\text {max }}$ at $\lambda=70$. At high network density, external interference is more dominant than residual self-interference. This trend remains until a certain point at which nodes' SIC capabilities become very low (i.e., $\chi_{\max }$ is very high). Under the BNE scheme, the HD action is more frequent than the FD action for all values of $\chi_{\max }$, as shown in Fig. 24. The reason for this behavior is that the network is very dense (i.e., $\lambda=70$ ), hence the HD action is more favorable. Fig. 25 shows the average per-link utility versus $\lambda$ at $\chi_{\max }=2 \times 10^{-4}$ for the four schemes. As the network density increases, the average per-link utility decreases due to the increase in the external interference. Since $\chi_{\max }$ is relatively high, the 'All HD' scheme outperforms the 'All FD' scheme. The average perlink utility under the BNE outperforms the other schemes.


Fig. 23. Average network utility versus $\chi_{\text {max }}(\lambda=70)$.


Fig. 24. Histogram of link actions under BNE versus $\chi_{\max }(\lambda=70)$.


Fig. 25. Average per-link utility versus $\lambda\left(\chi_{\max }=2 \times 10^{-4}\right)$.

## 7 Conclusions and Future Research

In this paper, we developed a novel Bayesian game-theoretic framework to study the coexistence problem between two FD-capable wireless links, where nodes have heterogeneous SIC capabilities. Although the throughput of a single link enhances significantly when operating in the FD mode, the additional interference caused (compared to the HD case) may limit its coexistence with a neighboring link. Our analysis revealed that the SIC capability of each link (which is the type of each player) has a double-threshold structure, i.e., the range of the SIC values can be divided into three regions. When the SIC capability is very good, operating in the FD mode strictly dominates the HD mode, whereas when the SIC capability is very poor, operating in the HD mode strictly dominates the FD mode. When the SIC capability is in the middle region, we derived the conditions on the probability distribution of the types of the other link under which HD (FD) strictly dominates FD (HD).

Our results corroborated that FD is not always the optimal mode when considering an FD-enabled wireless network. The optimal mode of a link depends on (i) the external interference it encounters from neighboring links
(which is a function of their transmission powers and channel gains) and (ii) its residual self-interference (which is a function of its SIC capability, its transmission power, and its channel gain). Given that the external interference will not be known a priori, a link relies on the probability distribution over the types of the other link in deriving its strategy. Our simulations demonstrated the impact of the residual self-interference and external interference on the BER.

To capture the interaction between more than two links, we studied the coexistence problem in a network setting, where multiple FD-enabled links with heterogeneous SIC capabilities are collocated in the same region. Specifically, we focused on the case where every player is mainly affected by a single dominating link. Under this assumption, we showed that different games in the network do not involve more than two players. In our network simulations, which is based on PPP, we showed that the network sum utility under the derived BNE outperforms other traditional schemes (e.g., all players play FD, HD, or flip a coin to select the action). We also studied the effect of the network density and players' SIC on players' sum utilities.

Different directions of future research exist. First, we will design practical distributed algorithms that enable multiple links to converge to the BNE without the need of a centralized node. Furthermore, we will investigate the optimization of the distributed algorithm to minimize the convergence time while maintaining low control overhead between nodes. Second, we will investigate a power control scheme to be used as part of the game, in addition to FD/HD mode selection. In this case, an additional dimension will be added to the game, where each player needs to select the optimal action (FD or HD) and the transmission power in each case.

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