Spectrum Bargaining: A Model for Competitive Sharing of Unlicensed Radio Spectrum

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Abstract—In this paper, we address the problem of dynamic channel access by a set of cognitive radio-enabled nodes, where each node acting in a selfish manner tries to access and use as many channels as possible, subject to interference constraints. We model the dynamic channel access problem as a modified Rubinstein–Ståhl bargaining game. In our model, each node (player) negotiates with the other nodes in the network reduce in a distributed manner to obtain an agreeable sharing rule of the available channels such that no two interfering nodes use the same channel. We solve the bargaining game by finding subgame perfect Nash equilibrium (SPNE) strategies of the game. First, we consider finite horizon version of the bargaining game and investigate its SPNE strategies, which allow each node to maximize its utility against the other nodes (opponents). We then extend these results to the infinite horizon bargaining game. Furthermore, we identify Pareto optimal equilibria of the game, which help enhance network throughput. We conduct simulations to study how the "self-gain" maximizing strategy of the players impact systemwide performance.

Index Terms—Dynamic spectrum access, spectrum sharing, interference, game theory, bargaining, finite and infinite horizon, subgame perfect nash equilibrium (SPNE).

I. INTRODUCTION

TRADITIONALLY, radio spectrum management has followed a 'command-and-control' approach- regulators like FCC allocate spectrum to specific services under restrictive licenses. The restrictions specify the technologies to be used and the services to be provided, thereby constraining the ability to make use of new technologies and the ability to redeploy the spectrum to higher valued uses. These limitations have motivated a paradigm shift from static spectrum allocation towards a more 'liberalized' notion of spectrum management in which secondary users can "borrow" idle spectrum from primary spectrum licensees, without causing harmful interference to the latter- a notion commonly referred to as dynamic spectrum access (DSA) or open spectrum access [43]. Cognitive radio [21], empowered by *S*oftware *D*efined *R*adio (SDR) [42], is poised to promote the efficient use of spectrum by adopting this open spectrum approach.

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In DSA systems, spectrum in not statically allocated to the secondary users (nodes). Each node has to *compete* for spectrum for communication purposes. Furthermore, the spectrum that a node uses is subject to interference constraints- nodes in close proximity interfere with each other and cannot use the same spectrum concurrently, while well separated nodes can reuse the same channel. Each node therefore has to use channels that are orthogonal from its interferers. If a node uses spectrum without coordinating with the others, then it may cause harmful interference and degrade overall spectrum usage. Clearly, from the above discussion, it becomes important to study the competition for spectrum among nodes in an interference aware context and investigate self-enforcing spectrum sharing strategies of the nodes.

In this paper, we consider nodes to behave in a selfish manner, i.e., the objective of each node is to maximize its utility by accessing and using as many channels as possible from the set of orthogonal channels not being used by any of the primary incumbents. The nodes in our model, for example, can correspond to broadcast access points deployed by competing wireless service providers. By using more channels each provider may intend to support more customers for maximizing its revenue. The channels that a node selects is, however, subject to the following constraint- nodes within the interference range of each other have to use orthogonal channels to minimize interference. Thus, the nodes will have to agree upon a *sharing rule* of the channels among themselves, i.e., each node will have to decide "how many" and "which" channels to use. In other words, *the channel access problem by a set of selfish nodes is inherently a bargaining game*, which has not been reflected in previous works [16], [27].

The fundamental question that we address in this paper is– *how many* and *which* channels should each node access to maximize its gain. Specifically, we model the problem of agreeing upon a sharing rule of the channels among the nodes as a Rubinstein-Ståhl [31], [36] bargaining game. In our model, each node "bargains" with the other nodes (opponents) in the network in a distributed manner regarding its "share" (*how many* and *which*) of the channels. Such distributed bargaining can be done, for example, using control channels [3]. Notice that, until the nodes agree upon the sharing rule, none of nodes can start data communication. Thus, "waiting" for the bargaining outcome also costs the nodes. We consider this cost by discounting future payoff of the nodes. This discounting represents the *patience* of the nodes in waiting for the bargaining outcome.

We solve the bargaining game by deriving Subgame Perfect Nash Equilbrium (SPNE) strategies of the players in the game.

The SPNE strategies that we derive comprise a set of strategies, such that, no player in no subgame can deviate from these strategies and thereby gain from his deviation. First, we investigate the finite horizon version of the game and identify its SPNE strategies. We then extend these results to the infinite horizon bargaining game. We propose computationally efficient algorithms to find the SPNE strategies of both the finite and the infinite horizon versions of the game. Furthermore, we identify Pareto optimal equilibria of the game for improving spectrum utilization. We also conduct simulations to study how the self-gain maximizing strategy of the players affect system wide performance.

The proposed spectrum bargaining is fundamentally different and much more difficult than the conventional Rubinstein-Ståhl bargaining. This is because of two primary reasons– (i) spectrum can be spatially reused concurrently; two conflicting players must not use the same channels simultaneously yet well-separated players can, and, (ii) players can only use whole channels, not fractional channels. We consider both constraints while analyzing the spectrum bargaining game.

The rest of the paper is organized as follows. Section II discusses related research in the area. Section III presents the system model and formally defines the bargaining game. Section IV investigates SPNE strategies in the finite horizon version of the game. These results are extended to the infinite horizon game in Section V. Section VI evaluates impact of the self-gain maximizing strategies of the players on overall system utility via simulations. Finally, Section VII concludes the paper.

II. Related Research

Coexistence of wireless systems that have to thrive by competing for spectrum has been studied in [11], [16], [19], [27], [20]. In [11], the authors consider coexistence of systems thriving on the same band, while we consider competition for spectrum in the more complicated scenario of an OFDM system where nodes can use multiple orthogonal channels for communication purposes. Moreover, the authors in [11] assume nodes in their model to be homogeneous. However, we study a heterogeneous environment and allow nodes to compete for spectrum in a differential manner. In [16], the authors model the competition among network operators who compete for spectrum. However, their framework is limited to a scenario where only two operators exist. In [19], the authors model the spectrum sharing problem as congestion games. However, they only consider the scenario where each node uses one channel. In [27], the authors use game theory to analyze strategies of cognitive radio nodes for accessing channels. Their solution approach is based on regret *minimization* of the nodes and uses an iterative learning algorithm using which nodes, that interact in a repeated game, can determine the channels to use. In contrast, we model the competition for spectrum among the nodes by considering the fact that each node will try to *maximize* its own benefit. Such modeling more aptly reflects non-cooperative interactions. Moreover, [27] assumes nodes to be homogeneous, unlike our model.

As far as spectrum trading is concerned, market based scenarios have been studied in [8], [9], [13], [34], [39], [23], [24]. In [8], a market in the form of a "bazaar" was introduced where infrastructure-based wide area wireless services are traded in a flexible manner and at any time scale. A general framework for spectrum trading based on auctions is proposed in [9], where an optimal auction mechanism called the generalized Branco's mechanism is introduced. Short-term secondary spectrum trading is considered in [13] where one seller and multiple buyers trade in a spectrum market with both guaranteed contracts and spot transactions. An economic framework was proposed in [34] where game theory was used for the dynamic spectrum allocation process and auctions were used for service pricing. All these works rely on a central entity, policy enforcer, or an auctioneer. However, in a competitive secondary spectrum market, there might not be any such central authority which makes the proposed bargaining model even more applicable as it is distributed in nature.

Spectrum sharing in wireless systems with an objective of maximizing overall system utility has been studied in [5], [6], [7], [29], [30], [32]. Techniques based on optimizing system utility primarily correspond to collaborative schemes among nodes usually deployed by the same wireless service provider. Buddhikot *et al.* in [5], [6], propose a spectrum access architecture via a regional spectrum broker. In [7], the authors propose a local bargaining approach for mobile ad-hoc networks, where users affected by mobility can form bargaining groups and adapt their spectrum assignment to approximate a new optimal assignment, instead of recomputing spectrum assignments for all users after each change in topology due to mobility. Their bargaining approach takes as input a previous spectrum assignment, and performs computations to adapt to recent topology changes. Nodes in their framework bargain to optimize a predefined system utility in contrast to ours where nodes bargain to maximize individual benefits. The authors in [29] formulate the problem of channel assignment, based on optimizing system utility, as a variant of the graph coloring problem by mapping channels into colors, and assigning them to users (nodes in the conflict graph of the network). They propose both a centralized allocation scheme, where a central server calculates an allocation assignment based on global knowledge, and a distributed approach, where devices negotiate local channel assignments towards a global optimization. In [30], the authors propose using a spectrum server to schedule the transmissions of a group of links sharing a common spectrum with an objective of optimizing network throughput. They assume that the spectrum server knows the link gains in the network. Using a linear programming approach, the server then finds an optimal schedule that maximizes the average sum rate subject to a minimum average rate constraint for each link. To utilize the bandwidth left unused in cellular systems (primary system), the authors in [32] propose the design of a secondary system in an overlay mode over the primary system. The secondary system operates in a non intrusive manner and does not interact with the primary cellular system. They design a Medium Access Control protocol that enables inter-operation of the primary-secondary systems.

Spectrum sharing by making nodes transmit at different power levels for minimizing interference has been studied in [10], [14], [15], [18], [26]. In [10], the authors consider power allocation strategies for radios operating in unlicensed bands. They model radio interaction as a two-player reputation based repeated game and use genetic algorithms to explore the

space of possible power allocation strategies. The authors in [14] design auction mechanisms for allocating power among a group of spread spectrum users who share the bandwidth with a licensed user. In these auctions, the spectrum owner charges for SINR and received power. The work in [15] considers a spectrum sharing problem in which each wireless transmitter can select a single channel from a set of available channels, along with the transmission power. In their scheme, users exchange price signals, that indicate the negative effect of interference at the receivers. Given this set of prices, each transmitter chooses a channel and power level to maximize its net benefit. The authors in [18] focus on decentralized power allocation strategies for sharing spectrum in a multi-user environment, where each user can communicate using multiple frequency bands. However, their model is restricted to a scenario where all users interfere with each other. In [26], the authors consider adaptive cognitive radio networks and study their convergence dynamics using Game Theory. In particular, they focus on distributed power control algorithms and investigate how they impact network complexity.

III. GAME FORMULATION

We first model the channel access problem as a *finite* horizon Rubinstein-Ståhl bargaining game. In this model, the game is played at most for a fixed number of stages. In Section V we will relax the finite horizon criteria and extend the concept to *infinite* horizon games. Before we begin with the finite horizon game formulation, let us first describe the system model that we consider.

A. System Model

We assume that N nodes (or players, denoted as $P_1, \ldots, P_N$) in a region are competing for a subset of M separate orthogonal spectrum bands (denoted as $C_1, \ldots, C_M$) not used by primary incumbents. The nodes, for example, can correspond to IEEE 802.22 base stations accessing spectrum to connect their subscribers units, or cognitive radio based IEEE 802.11 access points. Each node is equipped with cognitive radio and can communicate using multiple non-contiguous channels.

We assume that each node can successfully determine the presence of primary users on a channel and maintains a set of channels that it can use without affecting the operations of any primary user. Determination of the presence of primary users can either be done via sensing [1], [37], or by using the database recommended by FCC [22]. Interference among the nodes has been modeled using the pair wise binary matrix model [17]. We use the following notations to represent the two system parameters:

- **Interference constraint:** Let $I = \{I_{n,k} | I_{n,k} \in \{0, 1\}\}_{N \times N}$, be a N by N matrix, representing the interference constraint among nodes,

$$I_{n,k} = \begin{cases} 1 & \text{if node } n \text{ and } k \text{ conflict;} \\ 0 & \text{if node } n \text{ and } k \text{ do not conflict} \end{cases} \quad (1)$$

Note that, I is the adjacency matrix representing the conflict graph of the network. The problem of constructing the conflict graph has been extensively studied in literature, for example in [28], [35], [38], [41], and the references therein. The nodes can use techniques proposed in these earlier works to build the conflict graph.

- **Channel throughput:** Let $C = \{C_m : 1 \leq m \leq M\}$ be a M element array where C_m represents channel m. Thus, C represents the set of available channels. We consider a static interference environment without considering the impact of fast-scale channel fading, since all the nodes are static. Further, we assume that all channels are homogeneous.

The objective of each node is to acquire the maximum possible number of channels that are orthogonal from its interferers. As mentioned earlier, this can essentially be modeled as a bargaining problem. Thus, after determining the presence of primary users to form the set of available channels C, we consider that the nodes bargain among themselves to determine how to share the channels, with each node behaving in a selfish manner. If the nodes can agree upon a sharing rule, then they can use their agreed upon channel shares for communication purposes. Note that, depending on the primary user activity behavior, the bargaining model can correspond to a *finite* or *infinite* horizon game. If the primary user behavior changes relatively slowly, (e.g., a TV station being inactive throughout the night), the secondary nodes will have a considerably large amount of time for bargaining at their disposal [2]– in this case, the game tends toward having an infinite horizon. On the other hand, if the primary user activity changes on a relatively faster time scale, the secondary nodes will have relatively lesser time to bargain- in this case, the game tends toward having a finite horizon. In this paper, we address both the finite and infinite horizon versions of the spectrum bargaining game in Sections IV and V respectively.

B. Some Game Theoretic Definitions

Nash Equilibrium (NE): A NE is a set of strategies, one for each player, such that no player has an incentive to unilaterally change his strategy. Players are in equilibrium if no player can do better by unilaterally changing his or her strategy.

Pareto Optimal: Pareto optimality is a measure of efficiency. An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto optimal outcome cannot be improved upon without hurting at least one player.

Backward Induction: Backward induction is an iterative process for solving finite sequential games. First, one determines the optimal strategy of the player who makes the last move of the game. Then, the optimal action of the next-to-last moving player is determined taking the last player's action as given. This process continues backwards in time until all players' actions have been determined. Effectively, one determines the Nash equilibrium of each subgame of the original game.

Subgame Perfect Nash Equilibrium (SPNE): A SPNE is an equilibrium such that players' strategies constitute a Nash equilibrium in every subgame of the original game. It may be found by backward induction.

C. Finite Horizon Game Formulation

Given the conflict graph of a network, we now model the problem of channel access by the nodes (players) as a finite horizon bargaining game. In this game, N players (denoted as $P_1, \ldots, P_N$) must decide how to share the M available channels (denoted as $C_1, \ldots, C_M$) among them. The bargaining game proceeds in "time periods" in which one player proposes a sharing rule to the other players. Each of the other players can then either 'accept' or 'reject' the shares they have been respectively offered. The bargaining continues until a sharing rule has been accepted by all players *or* until the maximum number of allowable periods, T (numbered $0, \ldots, T-1$), has been reached (finite horizon game).

We consider that players make their offer in a round robin fashion. Specifically, P_i makes an offer in the following periods,

$$\begin{cases} kN + (i-1) & i \leq T, k \in [0, \lfloor (T-i)/N \rfloor] \\ \text{None} & i > T \end{cases} \quad (2)$$

where, k is an integer. The set of players receiving the offer of P_i is denoted as $P_{-i} = \{P_j : 1 \leq j \leq N, j \neq i\}$. When making an offer in period t, P_i's strategy is denoted by (x_i^t, x_{-i}^t), where $x_i^t \subset C$ is the set of channels demanded by P_i in period t. Also, $x_{-i}^t = \{x_j^t \subset C | 1 \leq j \leq N, j \neq i\}$ where x_j^t is the set of channels offered to $P_j \in P_{-i}$ in period t. Further, each player $P_j \in P_{-i}$ chooses some function $f_j^t : [0, |C|] \to \{\text{'accept', 'reject'}\}$ in period t, i.e, each $P_j \in P_{-i}$ chooses whether to accept or reject the offer depending on the number of channels he received, $|x_j^t|$.

To illustrate the game, let us consider that $N \leq T$[1]. In periods kN (for $k \in [0, \lfloor (T-1)/N \rfloor]$), player 1 ($P_1$) proposes a sharing rule (x_1^{kN}, x_{-1}^{kN}) to all players (including himself). After inspecting the offer, each player $P_j \in P_{-1}$ can either accept or reject the respective shares they have been offered. If all players in P_{-1} accept their respective shares, the game ends. However, if at least one player in P_{-1} rejects the share he has been offered by P_1 in period kN, then in period $kN + 1$ (for $k \in [0, \lfloor (T-2)/N \rfloor]$), player P_2 can propose a sharing rule $(x_2^{kN+1}, x_{-2}^{kN+1})$ that players in P_{-2} can accept or reject. If all players in P_{-2} accept their respective shares, the game ends. And so on, until an offer made by P_i is accepted by all players in P_{-i} or until the maximum number of allowable periods, T, has been reached. The game outlined above is clearly a finite horizon game of complete and perfect information.

Payoff: The outcome of the game can correspond to two different cases- all players agree upon a sharing rule of the channels within T periods or they fail to do so within the allocated time. Thus, to define the payoff of the players we need to study the following two cases.

- *An agreeable sharing rule is obtained within T periods:* If $\{x_i^t | 1 \leq i \leq N\}$ is accepted in period t, then the payoff of P_i is $\mathbb{R}_i = \delta_i^t |x_i^t|$, where $\delta_i \in [0, 1]$ is the discount factor of P_i (note, δ_i^t is δ_i raised to the power of t). The discount

factor represents the *delay cost* in achieving the bargaining outcome. Until the players agree upon a sharing rule, none of the players can start communication. Thus, a player values a channel more today than he values the same channel in a future period. This decrease in value of the channels represents the *dissatisfaction* of the players in being unable to start communication until the agreeable sharing rule is achieved. Also, note that, as the time delay between two bargaining periods decreases, the players become more patient, i.e, the discount factor of the players increases.

- *An agreeable sharing rule is not obtained within T periods:* This corresponds to the disagreement outcome of the game. Clearly, if the players are unable to agree upon a sharing rule, the payoff of each player would be *zero*.

D. Equilibrium Analysis

We will represent the strategy profile in period t as $\{(x_i^t, x_{-i}^t), f_{-i}^t\}$, where (x_i^t, x_{-i}^t) is the the sharing rule as proposed by P_i and $f_{-i}^t = \{f_j^t | 1 \leq j \leq N, j \neq i\}$, where f_j^t is the function used by $P_j \in P_{-i}$. If $f_j^t(|x_j^t|) = $ 'accept' $\forall j \neq i$, then each player gets his respective share as proposed in (x_i^t, x_{-i}^t). Otherwise all players get zero channels.

The strategy profile $\{(x_i^{T-1}, x_{-i}^{T-1}), f_{-i}^{T-1}\}$ is a NE in period $T-1$ (last period) if $f_j^{T-1}(|x_j^{T-1}|) = $ 'accept' $\forall j \neq i$ and there is no set $|y_i^{T-1}|$ such that $f_j^{T-1}(|y_j^{T-1}|) = $ 'accept' $\forall j \neq i$ that leads to the existence of a set $|y_i^{T-1}| > |x_i^{T-1}|$. Here, P_i does not have an incentive to unilaterally increase his demand, because that would be rejected by some $P_j \in P_{-i}$. Also, no $P_j \in P_{-i}$ would want to reject the share offered to him by P_i, since then he would get zero channels.

However, note that all NE's in the last period of the game need not be Pareto optimal. Our solution approach identifies and uses those NE's that are Pareto optimal to find the SPNE strategy of the players in the first period of the game using backward induction.

In the next section, we will study the SPNE of the finite horizon spectrum bargaining game. We will then extend these results for the infinite horizon version of the game in Section V.

IV. FINITE HORIZON BARGAINING GAME

We will now investigate the SPNE of the finite horizon bargaining game (of T periods), where each player bargains with the other players to agree upon a sharing rule of the channels. Finding SPNE involves two main steps– (1) finding equilibrium of the last period of the game, and, (2) finding equilibrium of the previous periods using backward induction.

A. Finding Last Period Equilibrium Strategies of the Players

According to the definition of NE in Section III-D, the following lemma presents the NE strategy for the players in the last stage of the game.

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[1]Assuming $N \leq T$ is for illustrating the game now. Our analysis holds for all N and T

LEMMA 1. P_i offers a sharing rule $(x_i^{T-1}, x_{-i}^{T-1})$, such that $|x_i^{T-1}|$ is maximized over all possible interference free allocations that assign at least 1 channel to all players in P_{-i}, and the players in P_{-i} accept all offers that give them at least 1 channel.

Proof: This is a NE because, no $P_j \in P_{-i}$ will have an incentive to reject their respective shares, since doing so will get them zero channels. Also, since P_i's share has been maximized, P_i will not have an incentive to demand a larger share of channels. ∎

Let us now see when P_i's share of channels gets maximized. Let us consider players in P_{-i} who are one hop away from P_i, i.e., neighbors of P_i in the conflict graph. Let they be denoted by P_{-i}^{Nbr}, where $P_{-i}^{Nbr} = \{P_j | P_j \in P_{-i} \text{ and } I_{i,j} = 1\}$. P_i will obviously demand all channels not allocated to any of his neighbors. Thus, we can write P_i's share of channels, x_i^{T-1}, as,

$$|x_i^{T-1}| = M - |\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}| \tag{3}$$

Clearly, $|x_i^{T-1}|$ will get maximized when $|\cup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ has its minimum value over all possible interference free allocations given that each player in P_{-i}^{Nbr} has to be given at least one non-interfering channel. In other words, P_i's share of channels is maximized in all those sharing rules where the number of *distinct* channels allocated to the players in P_{-i}^{Nbr} taken together has the least value over all possible allocations.

Let $P_{-i}^{\overline{Nbr}} = P_{-i} \backslash P_{-i}^{Nbr}$ denote the players in P_{-i} who are more than one hop away from P_i. Since spectrum can be reused concurrently by players more than one hop away from each other, the channel allocation of the players in $P_{-i}^{\overline{Nbr}}$ do not directly influence x_i^{T-1}. All that is required for P_i is to offer the players in $P_{-i}^{\overline{Nbr}}$ at least one non-interfering channel so that his offer is accepted.

From the above discussion, it can be said that P_i's strategy, $(x_i^{T-1}, x_{-i}^{T-1})$, corresponds to a NE when the following two conditions hold,

1) $|\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ is minimized over all possible interference free allocations, and,
2) Each player in P_{-i} gets *at least* one non-interfering channel. This condition is to ensure that the sharing rule offered by P_i is accepted, otherwise P_i's payoff will become zero.

Notice that there can be several NE strategy profiles, $(x_i^{T-1}, x_{-i}^{T-1})$, for P_i that maximizes $|x_i^{T-1}|$. However, not all of those will be *Pareto optimal*. It may be possible to improve the share of a player, $x_j^{T-1} \in x_{-i}^{T-1}$ without hurting any $x_k^{T-1}, k \in [1, N]$ and $k \neq j$. We need to find those NE's that are Pareto efficient to maximize spectrum utilization.

Algorithm 1 finds the Pareto efficient NE strategy $(x_i^{T-1}, x_{-i}^{T-1})$ of the player P_i making the offer in the last period of the game. The algorithm does three primary tasks.

1) **Find the last offerer:** First we need to find the player, P_i, who will make the offer in the last period. Algorithm 2 does this. It takes as input N and T and returns the ID of the player making the last offer.

Algorithm 1. Find Last Stage SPNE

Require: No. of Players, N; Interference Constraint, I; Set of Available Channels, C; Number of Periods, T
1: $i \leftarrow$ findLastOfferer(N,T)
2: Sort the players in P_{-i}^{Nbr} in non-increasing order according to their degree in g_{-i}^{Nbr}.
3: **for all** $P_j \in P_{-i}^{Nbr}$ **do**
4:     $x_j^{T-1} = \{C_m | C_m \notin \bigcup_{I_{j,q}=1} x_q^{T-1} \text{ and } C_k \in \bigcup_{I_{j,q}=1} x_q^{T-1} \forall k < m\}$
5: **end for**
6: $x_i^{T-1} \leftarrow \{C \backslash \bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}\}$
7: Sort the players in $P_{-i}^{\overline{Nbr}}$ in non-increasing order according to their degree in g_{-i}.
8: **for all** $P_j \in P_{-i}^{\overline{Nbr}}$ **do**
9:     $x_j^{T-1} = \{C_m | C_m \notin \bigcup_{I_{j,q}=1} x_q^{T-1} \text{ and } C_k \in \bigcup_{I_{j,q}=1} x_q^{T-1} \forall k < m\}$
10: **end for**
11: **while** true **do**
12:     assigned $\leftarrow$ false
13:     **for all** $P_j \in P_{-i}$ **do**
14:         **if** $\exists C_m : C_m \notin \bigcup_{I_{j,q}=1} x_q^{T-1} \text{ and } C_k \in \bigcup_{I_{j,q}=1} x_q^{T-1} \forall k < m$ **then**
15:             $x_j^{T-1} = \{x_j^{T-1} \bigcup C_m\}$
16:             assigned $\leftarrow$ true
17:         **end if**
18:     **end for**
19:     **if** !assigned **then**
20:         break
21:     **end if**
22: **end while**
23: Return $(x_i^{T-1}, x_{-i}^{T-1})$

Algorithm 2. findLastOfferer(N,T)

Require: Number of players, N; Number of Periods, $T \geq 1$
1: **if** $T \leq N$ **then**
2:     $i \leftarrow T$
3: **else**
4:     **if** $mod(T, N) == 0$ **then**
5:         $i \leftarrow N$
6:     **else**
7:         $i \leftarrow mod(T, N)$
8:     **end if**
9: **end if**
10: Return i

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2) **Find equilibrium strategy of P_i:** In order to find Pareto optimal NE strategy $(x_i^{T-1}, x_{-i}^{T-1})$ of P_i, where $|x_i^{T-1}|$ is maximized, our algorithm minimizes $|\cup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ such that each $P_j \in P_{-i}$ receives at least one channel and no $x_j^{T-1} \in x_{-i}^{T-1}$ can be improved without hurting any $x_k^{T-1}, k \in [1, N]$ and $k \neq j$.

First we will describe how the algorithm finds a NE $(x_i^{T-1}, x_{-i}^{T-1})$ that need not be Pareto efficient. Minimizing $|\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ is equivalent to the problem of coloring the subgraph induced by the players in P_{-i}^{Nbr} with the minimum number of colors. We will use degree ordered graph coloring for this purpose. Let the subgraph of the conflict graph induced by the players in P_{-i}^{Nbr} be g_{-i}^{Nbr} (need not be connected) and the subgraph induced by the players in P_{-i} be g_{-i}. g_{-i}^{Nbr} is also a subgraph of g_{-i}. To maximize $|x_i^{T-1}|$, g_{-i}^{Nbr} has to be colored with the least possible number of colors. Note that this is different from coloring g_{-i}, because a minimum color assignment of g_{-i} does not necessarily minimize the color assignment of g_{-i}^{Nbr}.

In line 2, the algorithm sorts the players in P_{-i}^{Nbr} in non-increasing order based on their degree in g_{-i}^{Nbr}. In lines 3 to 5, the algorithm considers each player $P_j \in P_{-i}^{Nbr}$ in non-increasing order of their degree, and assigns P_j the first channel in C that has not been assigned to any of P_j's neighbors. This process essentially intends to minimize $|\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$. After the for loop in lines 3 to 5 ends, we can thus assign P_i his maximizing share of $M - |\bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}|$ channels. This is done is line 6, which assigns,

$$x_i^{T-1} = \{C \setminus \bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{T-1}\} \tag{4}$$

Next, we are left with assigning a single channel to each player in $P_{-i}^{\overline{Nbr}}$ to find P_i's NE strategy $(x_i^{T-1}, x_{-i}^{T-1})$ that need not be Pareto optimal. To do this, the algorithm first sorts the players in $P_{-i}^{\overline{Nbr}}$ in non-increasing order according to their degree in g_{-i}. Then in lines 8 to 10, each player $P_j \in P_{-i}^{\overline{Nbr}}$ is considered in non-increasing order of their degree and assigned a channel that has not been assigned to any of P_j's neighbors.

$(x_i^{T-1}, x_{-i}^{T-1})$ obtained after the for loop in lines 8 to 10 ends is a NE strategy[2] for P_i even though it may not be *Pareto optimal*. Since each $P_j \in P_{-i}$ receives only one channel it may be possible to improve the share of some players in P_{-i} without decreasing the share of any other player. Notice that the share of P_i cannot possibly be improved further since it has already been optimized.

3) **Find Pareto optimal NE strategy of P_i:** Improvement of P_i's NE strategy $(x_i^{T-1}, x_{-i}^{T-1})$ obtained so far to get a Pareto optimal NE strategy is called Pareto improvement. This Pareto improvement is done by the while loop in lines 11 through 22. At each iteration of the while loop, the algorithm checks each player $P_j \in P_{-i}$ to see if a channel can be added to x_j^{T-1}. The while loop iterates till no more channels can be assigned to any player in P_{-i}. Clearly, after the while loop terminates, $(x_i^{T-1}, x_{-i}^{T-1})$ produced will correspond to a Pareto

optimal NE strategy of P_i. Also note that the Pareto improvements are done by trying to assign a single channel to a player $P_j \in P_{-i}$ at a time, instead of assigning all $C \setminus \{\{\bigcup_{I_{j,q}=1} x_q^{T-1}\} \bigcup x_j^{T-1}\}$ channels to P_j at the same time. This has been done to improve fairness.

B. Finding Equilibrium of the Previous Periods Using Backward Induction

Let P_i be the offerer in period t and P_l ($l \neq i$) be the offerer in period $t + 1$. Given the SPNE strategy of P_l in period $t + 1$, SPNE strategy of P_i in period t can be found based on the following fact- if a player $P_j \in P_{-i}$ gets $|x_j^{t+1}|$ channels in period $t + 1$, then in period t, P_j will accept any offer that gives him greater than equal to $\lceil \delta_j |x_j^{t+1}| \rceil$ channels. This is because x_j^{t+1} channels in period $t + 1$ is worth only $\delta_j |x_j^{t+1}|$ in period t to P_j. Thus P_j can be "satisfied" with only $\delta_j |x_j^{t+1}|$ channels in period t. However, since a player cannot get fractional channels, hence $\lceil \delta_j |x_j^{t+1}| \rceil$ channels has to be offered to P_j in period t. Formally, the following theorem presents the SPNE strategy in period t.

THEOREM 1. *The SPNE strategy in period t is comprised of the following.*

- **SPNE strategy of P_i:** *For each $P_j \in P_{-i}^{Nbr}$, P_i chooses a set of channels $c_j \subset x_j^{t+1}$ such that $|c_j| \leq (|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil)$ and for $C_s \in \bigcup_{P_j \in P_{-i}^{Nbr}} c_j$ it holds that $C_s \notin \bigcup_{P_j \in P_{-i}^{Nbr}} x_j^{t+1} \setminus c_j, \forall C_s \in \bigcup_{P_j \in P_{-i}^{Nbr}} c_j$. Also, $|\bigcup_{P_j \in P_{-i}^{Nbr}} c_j|$ should be the largest such set possible so that $x_i^t = \{x_i^{t+1} \bigcup \{\bigcup_{P_j \in P_{-i}^{Nbr}} c_j\}\}$ is maximized. Each $P_j \in P_{-i}^{Nbr}$ is offered the set of channels $x_j^t = x_j^{t+1} \setminus c_j$. In other words, P_i offers at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels to each $P_j \in P_{-i}^{Nbr}$, taking at most $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from each $P_j \in P_{-i}^{Nbr}$ such that x_i^t is maximized over all possible interference free allocations that allows P_i to take at most $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ from each $P_j \in P_{-i}^{Nbr}$.*
- **SPNE strategy of $P_j \in P_{-i}$:** *Each $P_j \in P_{-i}$ accepts all offers in which they get at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels.*

Proof: Clearly, no player will have an incentive to unilaterally deviate from his strategy. If P_i makes a larger demand of channels than x_i^t defined above, then some $P_j \in P_{-i}^{Nbr}$ has to be given a smaller share of channels than $\lceil \delta_j |x_j^{t+1}| \rceil$ in period t, and thus P_i's offer will be rejected. If rejected, in no subsequent period can P_i hope to get a share of channels which in period t is worth more than $|x_i^t|$[3]. Also, no player $P_j \in P_{-i}$ in period t can hope to get a share of channels in any subsequent period which in period t is worth more than $\lceil \delta_j |x_j^{t+1}| \rceil$. Thus, the above mentioned strategy of the players comprises a SPNE in period t. In other words, *no player can deviate from his above mentioned strategy in period t and subsequently gain from his deviation in any sub-game starting from period t.*

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[2] $|x_i^{T-1}|$ has been maximized, and all players in P_{-i} receives a channel each – so no $P_j \in P_{-i}$ has an incentive to reject his share and also P_i does not have an incentive to demand more channels

[3] Recall that if a player P_j, $j \in [1, N]$ gets the set of channels x_j^{t+2} in period $t + 2$, then in period t it is worth only $\delta_j^2 |x_j^{t+2}|$ to P_j.

Algorithm 3. Find SPNE by backward induction

Require: No. of Players, N; Interference Constraint, I; Set of Available Channels, C; Number of Periods, T

1: $(x_l^{T-1}, x_{-l}^{T-1}) \leftarrow$ Find Last Stage SPNE(N,I,C,T)
2: **for** $t = T - 2$ to 0 **do**
3:    **if** $l = 1$ **then**
4:      $i \leftarrow N$
5:    **else**
6:      $i \leftarrow l - 1$
7:    **end if**
8:    $\widehat{P}_{-i}^{Nbr} \leftarrow \{\phi\}$
9:    **for all** $P_j \in P_{-i}^{Nbr}$ **do**
10:      $s \leftarrow |x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$
11:      **if** $s > 0$ **then**
12:        $Q_j \leftarrow \mathcal{P}_s(x_j^{t+1}) \backslash \mathcal{P}_{s-1}(x_j^{t+1})$
13:        $\widehat{P}_{-i}^{Nbr} \leftarrow \widehat{P}_{-i}^{Nbr} \bigcup P_j$
14:      **end if**
15:    **end for**
16:    **if** $\widehat{P}_{-i}^{Nbr}$ is null **then**
17:      $x_i^t \leftarrow x_i^{t+1}$
18:      $x_j^t \leftarrow x_j^{t+1}, \forall P_j \in P_{-i}$
19:      $l \leftarrow i$
20:      continue
21:    **else**
22:      $Q \leftarrow \prod_{j=1}^N Q_j : P_j \in \widehat{P}_{-i}^{Nbr}$
23:    **end if**
24:    **for** $r = 1$ to $r = |Q|$ **do**
25:      $index \leftarrow 1$
26:      $S \leftarrow \{\bigcup_{P_j \in P_{-i}^{Nbr} \backslash \widehat{P}_{-i}^{Nbr}} x_j^{t+1}\}$
27:      **for all** $j = 1$ to $j = N : P_j \in \widehat{P}_{-i}^{Nbr}$ **do**
28:        $s \leftarrow |x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$
29:        $c_j^r \leftarrow_{index}^{index+s-1} q_r$ {Elements $index$ to $index+s-1$ of set q_r}
30:        $S \leftarrow S \bigcup \{x_j^{t+1} \backslash c_j^r\}$
31:        $index \leftarrow index + s$
32:      **end for**
33:      profit$(q_r) \leftarrow \{q_r \backslash S\}$
34:    **end for**
35:    Select $q_m \in Q : |\text{profit}(q_m)| = \max_{q_r \in Q; 1 \leq r \leq |Q|} (|\text{profit}(q_r)|)$
36:    $x_i^t \leftarrow x_i^{t+1} \bigcup \text{profit}(q_m)$
37:    $x_j^t \leftarrow x_j^{t+1} \backslash \text{profit}(q_m), \forall P_j \in \widehat{P}_{-i}^{Nbr}$
38:    $x_j^t \leftarrow x_j^{t+1}, \forall P_j \in P_{-i} \backslash \widehat{P}_{-i}^{Nbr}$
39:    **while** true **do**
40:      $assigned \leftarrow$ false
41:      **for all** $P_j \in P_{-i}$ **do**
42:        **if** $\exists C_m \in C : C_m \notin \bigcup_{I_{j,q}=1} x_q$ and $C_k \in \bigcup_{I_{j,q}=1} x_q$ $\forall k < m$ **then**
43:          $x_j^t = \{x_j^t \bigcup C_m\}$
44:          $assigned \leftarrow$ true
45:        **end if**
46:      **end for**
47:      **if** !assigned **then**
48:        break
49:      **end if**
50:    **end while**
51:    $l \leftarrow i$
52: **end for**
53: Return (x_1^0, x_{-1}^0)

We now show the existence of the SPNE sharing rule (x_i^t, x_{-i}^t) in period t. Consider first the set of channels x_j^t offered to $P_j \in P_{-i}^{Nbr}$. Note, that $x_j^t \subset x_j^{t+1}$. Therefore, x_j^t exists. Next, note that at equilibrium, $x_i^t = C \backslash \bigcup_{P_j \in P_{-i}^{Nbr}} x_j^t$. Therefore, since x_j^t exists $\forall P_j \in P_{-i}^{Nbr}$, the set x_i^t exists. ∎

Algorithm 3 finds the SPNE strategy of the offerer $P_i, i \in [1, N]$ in period t given the equilibrium strategy of the offerer $P_l, l \in [1, N], l \neq i$ in period $t + 1$ to finally find the SPNE strategy (x_1^0, x_{-1}^0) of P_1 in the first period of the game. The algorithm first invokes Algorithm 1 to find the equilibrium strategy, $(x_l^{T-1}, x_{-l}^{T-1})$, of offerer P_l in period $T - 1$ (last period) and works in iterations of decreasing period number, using backward induction to find the SPNE strategy of the offerer in the respective period at each iteration. The algorithm finally outputs the SPNE strategy (x_1^0, x_{-1}^0) of offerer P_1 in the *first* period of the game. As discussed earlier, this is such a strategy that no $P_j \in P_{-1}$ can gain in any sub-game by rejecting his respective share in x_{-1}^0. Also P_1's share, x_1^0, is maximized so that P_1 does not have an incentive to demand a larger share of channels. Let us now delve into the details of Algorithm 3. We will explain how the algorithm finds the equilibrium strategy (x_i^t, x_{-i}^t) of offerer P_i in period t given the equilibrium strategy $(x_l^{t+1}, x_{-l}^{t+1})$ of offerer P_l ($l \neq i$) in period $t + 1$. Algorithm 3 does the following tasks:

1) **Find offerer P_i in period t:** First the algorithm finds the offerer P_i in period t. This is done in lines 3 to 7.
2) **Find SPNE strategy for P_i in period t:** P_i's SPNE strategy in period t will correspond to P_i taking the maximum possible number of channels from his neighbors in order to maximize his share of channels in period t. In order to maximize the number of channels that P_i can acquire from his neighbors in P_{-i}^{Nbr}, P_i will have to consider all *interference free* allocations that allow him to take at most $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from $P_j \in P_{-i}^{Nbr}$ and use the one that allows P_i to take the maximum number of channels from his neighbors. Steps 8–38 basically does this. Let us look at these steps in more detail.

Note that, P_i can potentially take channels only from those $P_j \in P_{-i}^{Nbr}$ for whom $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil > 0$. Let $\widehat{P}_{-i}^{Nbr} \subset P_{-i}^{Nbr}$ be the set of neighbors of P_i satisfying this criteria. The for loop in lines 9 to 15 finds the set $\widehat{P}_{-i}^{Nbr}$ and also generates the set Q_j for each $P_j \in \widehat{P}_{-i}^{Nbr}$, where Q_j (defined as $\mathcal{P}_s(x_j^{t+1}) \backslash \mathcal{P}_{s-1}(x_j^{t+1}))$[4] is the set of all

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[4]Here, s denotes the number of channels that can be taken by P_i from $P_j \in P_{-i}^{Nbr}$ (which is computed in Line 10). $\mathcal{P}_s(x_j^{t+1})$ and $\mathcal{P}_{s-1}(x_j^{t+1})$ denote the set of all subsets of x_j^{t+1} of cardinality less than equal to s and $s - 1$, respectively. Note, this implies that $\mathcal{P}_s(x_j^{t+1}) \backslash \mathcal{P}_{s-1}(x_j^{t+1})$ is the set of all subsets of x_j^{t+1} of cardinality s (which is computed in Line 12).

subsets of x_j^{t+1} with cardinality $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$. Note that, it is possible that the set $\widehat{P}_{-i}^{Nbr}$ is null. In this case P_i cannot take channels from any of his neighbors. Thus, the SPNE strategy for P_i will be to demand the set of channels, $x_i^t = x_i^{t+1}$, and offer each $P_j \in P_{-i}$ the set of channels $x_j^t = x_j^{t+1}$. The case of $\widehat{P}_{-i}^{Nbr}$ being null is taken care of in lines 16 to 20 at the end of which the algorithm *continues* onto the next iteration to find the strategy of the offerer in period $t - 1$.

For the case when $\widehat{P}_{-i}^{Nbr}$ is not null, we define set Q as the cartesian product of all Q_j for $P_j \in \widehat{P}_{-i}^{Nbr}$. Thus, element $q_r \in Q, r \in [1, |Q|]$, is a set of channels that P_i can acquire from his neighbors in $\widehat{P}_{-i}^{Nbr}$ taken together, taking $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from neighbor $P_j \in \widehat{P}_{-i}^{Nbr}$. Set Q has all such combination of channels that P_i can take from his neighbors in $\widehat{P}_{-i}^{Nbr}$. Notice that in line 22, the cartesian product of all Q_j for $P_j \in \widehat{P}_{-i}^{Nbr}$ considers the Q_j's in ascending order of their subscripts. According to our definition of Q and rules of cartesian product, if P_j is the lowest numbered player in $\widehat{P}_{-i}^{Nbr}$ [5], then the first $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels of $q_r \in Q$ belong to $P_j \in \widehat{P}_{-i}^{Nbr}$. Likewise, if P_k is the second lowest numbered player in $\widehat{P}_{-i}^{Nbr}$ then the next $|x_k^{t+1}| - \lceil \delta_k |x_k^{t+1}| \rceil$ channels belong to $P_k \in \widehat{P}_{-i}^{Nbr}$. And so on. Let $c_j^r \subset q_r$ be the set of channels belonging to $P_j \in \widehat{P}_{-i}^{Nbr}$. Notice that P_i can use a channel $C_k \in q_r$, iff $C_k \notin \{\bigcup_{P_j \in \widehat{P}_{-i}^{Nbr}} \{x_j^{t+1} \backslash c_j^r\}\} \bigcup \{\bigcup_{P_j \in P_{-i}^{Nbr} \backslash \widehat{P}_{-i}^{Nbr}} x_j^{t+1}\}$. In other words, P_i can use a channel $C_k \in q_r$ if and only if *no* $P_j \in \widehat{P}_{-i}^{Nbr}$ has C_k *after* the set of channels c_j^r has been taken from P_j *and* neither does any $P_j \in P_{-i}^{Nbr} \backslash \widehat{P}_{-i}^{Nbr}$ have the channel C_k. Let profit$(q_r) \subset q_r$ be the set of channels that P_i can use from among the channels in q_r. Thus,

$$\text{profit}(q_r) = q_r \backslash \{\{\bigcup_{P_j \in \widehat{P}_{-i}^{Nbr}} \{x_j^{t+1} \backslash c_j^r\}\} \bigcup \{\bigcup_{P_j \in P_{-i}^{Nbr} \backslash \widehat{P}_{-i}^{Nbr}} x_j^{t+1}\}\} \quad (5)$$

where, $c_j^r \subset q_r$ is the set of channels belonging to $P_j \in \widehat{P}_{-i}^{Nbr}$. The for loop in lines 24 to 34 finds the set profit$(q_r) \subset q_r$ for $q_r \in Q, \forall r \in [1, |Q|]$. Trying to maximize the number of channels P_i can acquire from his neighbors, P_i will choose set $q_m \in Q, m \in [1, |Q|]$ such that,

$$|\text{profit}(q_m)| = \max_{q_r \in Q; 1 \le r \le |Q|} (|\text{profit}(q_r)|) \quad (6)$$

and take the set of channels profit(q_m) from his neighbors. Thus, in period t, P_i's share of channels will be (line 36),

$$x_i^t \leftarrow x_i^{t+1} \bigcup \text{profit}(q_m) \quad (7)$$

Also, clearly in period t, each $P_j \in \widehat{P}_{-i}^{Nbr}$ will be left with the set of channels $\{x_j^{t+1} \backslash \text{profit}(q_m)\}$. Since each $P_j \in \widehat{P}_{-i}^{Nbr}$ will have at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels [6] in period t, no P_j will have an incentive to reject his share of (line 37),

$$x_j^t \leftarrow x_j^{t+1} \backslash \text{profit}(q_m), \quad \forall P_j \in \widehat{P}_{-i}^{Nbr} \quad (8)$$

All other players, i.e., $P_j \in P_{-i} \backslash \widehat{P}_{-i}^{Nbr}$ [7], can have the same share of channels in period t as they had in period $t + 1$, and thus no P_j will not have an incentive to reject their share of (line 38),

$$x_j^t \leftarrow x_j^{t+1}; \forall P_j \in P_{-i} \backslash \widehat{P}_{-i}^{Nbr} \quad (9)$$

The strategy (x_i^t, x_{-i}^t) obtained for P_i is a SPNE strategy for P_i in period t. Clearly, no $P_j \in P_{-i}$ can gain in any sub-game (play after period t) by rejecting their respective shares in x_{-i}^t. Also, P_i cannot make a "successful" demand of a larger share of channels than x_i^t, since the number of channels that P_i can take from his neighbors has been optimized over all possible interference free allocations that allows P_i to take at most $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from $P_j \in P_{-i}^{Nbr}$. However, (x_i^t, x_{-i}^t) strategy for P_i obtained so far *may not be Pareto optimal*. It may be possible to improve the share of some players in P_{-i} without decreasing the share of any player. We deal with this next.

3) **Find Pareto optimal SPNE strategy for P_i:** Pareto improvement of P_i's SPNE strategy, (x_i^t, x_{-i}^t), obtained so far to obtain a Pareto optimal strategy for P_i is done by the while loop in steps 39 through 50. At each iteration of the while loop, the algorithm checks each player $P_j \in P_{-i}$ to see if a channel can be added to x_j^t. The while loop iterates till no more channels can be assigned to any player in P_{-i}. Clearly, after the while loop terminates, (x_i^t, x_{-i}^t) produced will correspond to a Pareto optimal SPNE strategy for P_i in period t.

When Algorithm 3 terminates, it finds a SPNE strategy, (x_1^0, x_{-1}^0), for P_1 in the first period, such that P_1 cannot make a larger demand of channels than x_1^0 that will be accepted by all players in P_{-1}. If rejected, P_1 cannot hope to get a share of channels in any subsequent period which in period 0 is worth more than $|x_1^0|$. Also, no player in P_{-1} can hope to get a share of channels in any subsequent period which in period 0 is worth more than his respective share in x_{-1}^0. Thus, the players in P_{-1} does not have an incentive to reject their share in x_{-1}^0. Formally, the correctness of Algorithm 3 in finding a SPNE can be proved by showing that the following invariant is true for all iterations– "At iteration t, (x_i^t, x_{-i}^t) is the SPNE of period t". This can be shown as follows. Given the SPNE $(x_i^{t+1}, x_{-i}^{t+1})$ of period

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[5] Let $Q_1 = \{\{C_1, C_2\}, \{C_3, C_4\}\}$ and $Q_2 = \{\{C_3, C_5\}, \{C_6, C_7\}\}$. Then $Q = Q_1 \times Q_2 = \{\{C_1, C_2, C_3, C_5\}, \{C_1, C_2, C_6, C_7\}, \{C_3, C_4, C_3, C_5\}, \{C_3, C_4, C_6, C_7\}\}$. For any $q \in Q$, the first two elements of q belong to Q_1 and the next two elements belong to Q_2. Also we define Q to be $Q_1 \times Q_2$ and not $Q_2 \times Q_1$

[6] $q_r \in Q$ has been computed taking $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ from $P_j \in \widehat{P}_{-i}^{Nbr}$. Thus, each $P_j \in \widehat{P}_{-i}^{Nbr}$ will have at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels in the share offered to them in period t.

[7] Note that this includes neighbors of P_i from whom P_i could not take any channels ($P_{-i}^{Nbr} \backslash \widehat{P}_{-i}^{Nbr}$), as well as players who are more than a hop away from P_i ($\overline{P_{-i}^{Nbr}}$)

Algorithm 4. Find SPNE strategy of offerer P_i in period t

1: $\mathcal{S} \leftarrow \{\phi\}$
2: **for all** $C_m \in C : C_m \notin \cup_{P_j \in P_{-i}^{Nbr} \setminus \widehat{P}_{-i}^{Nbr}} x_j^{t+1}$ **do**
3:     $s \leftarrow \sum_{P_j \in \widehat{P}_{-i}^{Nbr}} |C_m \cap x_j^{t+1}|$
4:     **if** $s > 0$ **then**
5:       $\mathcal{S} \leftarrow \mathcal{S} \cup C_m^s$
6:     **end if**
7: **end for**
8: Sort the channels in $\mathcal{S}$ in non-decreasing order according to their superscripts in $\mathcal{S}$.
9: Mark all $P_j \in \widehat{P}_{-i}^{Nbr}$ as available.
10: $r_j \leftarrow |x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil \; \forall P_j \in \widehat{P}_{-i}^{Nbr}$
11: $x_j^t \leftarrow \{x_j^{t+1}\} \; \forall j \in [1, N]$
12: **for all** $C_m \in \mathcal{S}$ **do**
13:     **if** <u>all</u> $P_j \in \widehat{P}_{-i}^{Nbr}$ for which $|x_j^t \cap C_m| = 1$ is marked available **then**
14:       $r_j \leftarrow r_j - 1 \; \forall P_j \in \widehat{P}_{-i}^{Nbr} : |x_j^t \cap C_m| = 1$
15:       $x_j^t \leftarrow x_j^t \setminus C_m \; \forall P_j \in \widehat{P}_{-i}^{Nbr} : |x_j^t \cap C_m| = 1$
16:       $x_i^t \leftarrow x_i^t \cup C_m$
17:       Mark all $P_j \in \widehat{P}_{-i}^{Nbr}$ as unavailable for which $r_j = 0$.
18:     **end if**
19: **end for**

Fig. 1. Pareto optimal SPNE of the respective offerer in different periods.

P_{-i} without decreasing the share of any player. The Pareto optimal SPNE strategy of P_i can be found by a while loop similar to the one in lines 39 to 50 of Algorithm 3.

It can be easily verified that the worst case complexity of finding the SPNE strategy of offerer P_i in period t is $O(M^2 D)$, where D is the highest degree of a node in the conflict graph.

C. An Illustrative Example

Figure 1 shows an example of how to find the SPNE strategy (x_1^0, x_{-1}^0) of P_1 in the first period of the game. We consider a network of 6 nodes. The graphs in Figure 1 depict the conflict graph of the network. The number of channels available, M, has been assumed to be 5. Each node has a discount factor of 0.5. The game is played for 6 periods. The channels assigned to the nodes in each period has been shown in brackets beside the node.

Figure 1(a) shows the pareto optimal NE strategy of P_6 in the last period of the game as obtained by using Algorithm 1. The offerer in this period is P_6. First, a NE strategy of P_6 in the last stage (which need not be pareto optimal) is found using lines 2 to 10 of the algorithm. In lines 2 to 6, P_6 colors his neighbors (P_2 and P_3) with the least possible number of colors (channels) and keeps rest of the channels for himself (thereby maximizing his share). In lines 7 to 10, the players in ($P_{-6}^{\overline{Nbr}}$) (i.e, P_1, P_4 and P_5) are given a channel each by graph coloring them. This is done by considering P_1, P_4 and P_5 in non-increasing order of their degree in the subgraph induced by P_1, P_2, P_3, P_4 and P_5. Thus, P_1 is considered first[8] and gets C_2, next P_5 gets C_3 and finally P_4 gets channel C_1. Clearly, the channel assignment obtained so far corresponds to a NE strategy of P_1 in the last stage, but one that may not be pareto

$t + 1$, iteration t finds the channel share (x_i^t, x_{-i}^t), such that, x_i^t has the maximum share of channels that the offerer P_i can keep while offering at least $\lceil \delta_j |x_j^{t+1}| \rceil$ channels to every $P_j \in P_{-i}$. Therefore, based on Theorem 1, (x_i^t, x_{-i}^t) is the SPNE of period t of the game, implying that the invariant is preserved. When the iterations terminate at $t = 0$, (x_i^0, x_{-i}^0) is the SPNE spectrum share at period 0 of the game, which, by definition, is the SPNE share of the spectrum bargaining game.

1) A more efficient implementation: Though Algorithm 3 lays down the basic idea of finding SPNE strategies of the spectrum bargaining game using backward induction, it checks all combinations of channels that allow an offerer P_i in period t to take $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels from neighbor $P_j \in \widehat{P}_{-i}^{Nbr}$. To do away with checking all such combinations, Algorithm 4 presents a more computationally efficient procedure for finding the SPNE strategy of offerer P_i in period t (that need not be Pareto optimal).

The critical task is to find the maximum set of channels that offerer P_i can acquire in period t. Recall that, P_i can potentially take channels only from those $P_j \in P_{-i}^{Nbr}$ for whom $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil > 0$ (denoted as $\widehat{P}_{-i}^{Nbr}$). Keeping this in mind, Algorithm 4 finds the *maximum* set of channels that P_i can *take* from this neighbors. The basic idea of the algorithm is to sort the channels that the players in $\widehat{P}_{-i}^{Nbr}$ has (in period $t + 1$) in ascending order based on the number of players that possess each channel. The algorithm then considers taking each channel in this ascending order, ensuring that no more than $|x_j^{t+1}| - \lceil \delta_j |x_j^{t+1}| \rceil$ channels is taken from each $P_j \in \widehat{P}_{-i}^{Nbr}$.

Notice that the strategy (x_i^t, x_{-i}^t) found by Algorithm 4 for offerer P_i in period t need not yield a Pareto optimal equilibrium. It may be possible to improve the share of some players in

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[8] We consider that, if two nodes have the same degree then the lower numbered node is considered first

optimal, since the shares of some players (P_1, P_4 and P_5) can be improved without hurting the share of any other player.

The pareto optimal NE strategy of P_6 is obtained using lines 11 to 22 of Algorithm 1. This is done by considering the players in P_{-6} and checking to see if more channels can be assigned to the player. In the first iteration of the while loop, P_1 receives C_4, P_2 and P_3 does not get any more channels, P_4 gets C_3 and P_5 gets C_5. In the second iteration of the while loop only P_4 gets C_5. The channel assignment obtained now is shown in Figure 1(a), and corresponds to the pareto optimal NE strategy of P_6 in the last stage of the game.

We will exemplify the concept of finding the SPNE strategy of an offerer P_i in period t from the SPNE strategy of offerer P_l ($l \neq i$) in period $t + 1$ (refer Algorithm 4) by showing how to find the SPNE strategy of the offerer P_3 in period 2 from the SPNE strategy of P_4 in period 3. Note that $P_{-3}^{Nbr} = \{P_1, P_5, P_6\}$. The four channels that P_6 has in period 3 is equivalent to having $\lceil 4 \times 0.5 \rceil = 2$ channels in period 2. Thus, P_3 can potentially take 2 channels from P_6. Similarly, the 3 channels that P_5 has in period 3 is equivalent to having $\lceil 3 \times 0.5 \rceil = 2$ channels in period 2. Thus, P_3 can potentially take 1 channel from P_5. However, P_3 will not be able to take any channel from P_1, since the latter has only one channel in the previous period. In fact, this also implies that P_3 will not be able to take channel 4 (due to interference constraints). Thus $\widehat{P}_{-3}^{Nbr} = \{P_6, P_5\}$. Now, Algorithm 4 will sort the channels that P_3 can potentially take from this neighbors in $\widehat{P}_{-3}^{Nbr}$ in ascending order based on the number of players in $\widehat{P}_{-3}^{Nbr}$ that possess each channel (line 8 Algorithm 4). This yields the set $S = \{C_2, C_3, C_5\}$. First, C_2 is considered. Since the channel is possessed by both players P_5 and P_6 and a channel can be taken from both the players, P_3 takes channel C_2 from P_5 and P_6. Next, channel C_3 is considered, which is also possessed by both players P_5 and P_6. However, since no more channels can now be taken from P_5, P_3 cannot take C_3. Similarly, C_5 cannot be taken also. Thus, P_3 only takes C_2 from both P_5 and P_6, making $x_3^2 = \{1, 2\}$. It can be easily seen that the maximum number of channels that P_3 can acquire from his neighbors is indeed 1.

The channel assignment obtained so far is a SPNE strategy for P_3 in period 2. However, it may not be pareto optimal. To obtain the pareto optimal SPNE strategy of P_3, a while loop of the form discussed earlier can be used which would assign C_2 to P_2. The channel assignment obtained subsequently would correspond to the pareto optimal SPNE strategy of P_3 in period 2 of the game. This is shown in Figure 1(d).

Following the same line of reasoning, we finally obtain the SPNE strategy (x_1^0, x_{-1}^0) of P_1 in the first period of the game. This strategy of P_1 is shown in Figure 1(f).

V. INFINITE HORIZON BARGAINING GAME

In this section, we consider the scenario where the spectrum bargaining game has an infinite horizon, i.e., players can go on bargaining until all players can agree on a sharing rule of the channels. To study the infinite horizon game, we will first extend the definition of the finite horizon game defined in Section III. Specifically, in the infinite horizon game, each P_i for $i \in [1, N]$ makes an offer in periods $kN + (i - 1)$ where

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Algorithm 5. Find Infinite Horizon SPNE

Require: Number of players, N; Interference Constraint, I; Set of Available Channels, C; Number of Periods, T;
1: $(x_1^0, x_{-1}^0)_j^{T-1} \leftarrow \{\phi\}, \forall j \in [1, N]$
2: **while** true **do**
3:     $(x_1^0, x_{-1}^0)^T \leftarrow$ Find SPNE by B.I(N, I, C, T)
4:     **if** $|(x_1^0, x_{-1}^0)_j^T| = |(x_1^0, x_{-1}^0)_j^{T-1}|, \forall j \in [1, N]$ **then**
5:        break
6:     **else**
7:        $T \leftarrow T + 1$
8:     **end if**
9: **end while**
10: Return $(x_1^0, x_{-1}^0)^T$

$k \in \{0, 1, 2, \ldots\}$. We will denote the SPNE strategy of P_1 in the first period of a T period finite horizon game as $(x_1^0, x_{-1}^0)^T$. Also, let $(x_1^0, x_{-1}^0)_j^T$ denote set x_j^0 in $(x_1^0, x_{-1}^0)^T$ for $j \in [1, N]$. Recall that $(x_1^0, x_{-1}^0) = (x_1^0, \{x_2^0, x_3^0, \ldots, x_N^0\})$.

Our solution of the *infinite* horizon game is based on the following fact- for the *finite* horizon game, there exists a period T such that the number of channels each player receives in the SPNE strategy of P_1, $(x_1^0, x_{-1}^0)^T$, of a T period game, is *equal to* the number of channels each player receives in the SPNE strategy of P_1, $(x_1^0, x_{-1}^0)^{T'}$, in a T' period game, for all $T' > T$. In other words, $\exists T$ such that for all $T' > T$ we have $|(x_1^0, x_{-1}^0)_j^{T'}| = |(x_1^0, x_{-1}^0)_j^T|, \forall j \in [1, N]$. We will show this via simulations in Figures 2(a) and 2(b). Figure 2(a) shows the SPNE strategy $(x_1^0, x_{-1}^0)^T$ of a 6 player finite horizon game with varying number of periods, T. The conflict graph has been randomly generated. The number of channels available, M, has been taken to be 8 and the discount factor of all players is 0.7. As can be clearly seen from the figure, for $T > 6$, $|(x_1^0, x_{-1}^0)_j^T| = |(x_1^0, x_{-1}^0)_j^6|, \forall j \in [1, N]$. For the game in Figure 2(b), the number of players have been taken to be 12. The conflict graph again has been randomly generated. The number of channels available, M, has been taken to be 15, and the discount factor of all players is 0.75. As can be seen from Figure 2(b), the SPNE spectrum shares of the players converge for games having 15 periods or more. Thus, based on the above discussion, to find the SPNE strategy, $(x_1^0, x_{-1}^0)^\infty$, of the infinite horizon game, we simply need to find the SPNE strategy, $(x_1^0, x_{-1}^0)^T$, of a finite horizon game of T periods, such that for all $T' > T$ we have $|(x_1^0, x_{-1}^0)_j^{T'}| = |(x_1^0, x_{-1}^0)_j^T|, \forall j \in [1, N]$. Based on this, Algorithm 5, gives the procedure that P_1 will invoke to find his SPNE strategy, $(x_1^0, x_{-1}^0)^\infty$, in the infinite horizon game.

A. Discussions

1) Value of T, the Starting Period: Algorithm 5 finds the SPNE of P_1 in the first period, starting from a T period game, until it finds a T' period game, such that, $|(x_1^0, x_{-1}^0)_j^{T'}| = |(x_1^0, x_{-1}^0)_j^{T'-1}|, \forall j \in [1, N]$. The algorithm then outputs the SPNE of P_1 in the infinite horizon game as $(x_1^0, x_{-1}^0)^{T'}$. The period T' at which the SPNE strategy of P_1 converges depends
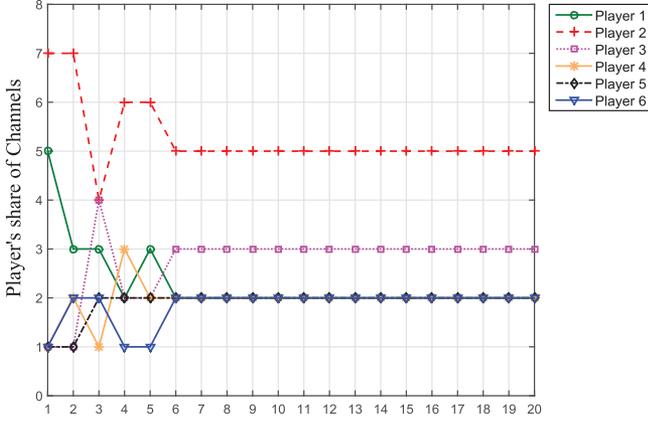
(a) Pareto optimal SPNE strategy $(x_1^0, x_{-1}^0)^T$ of a 6-player finite horizon game with varying number of periods (T). Here $M = 8$, and $\delta_i = 0.7 \; \forall i$.
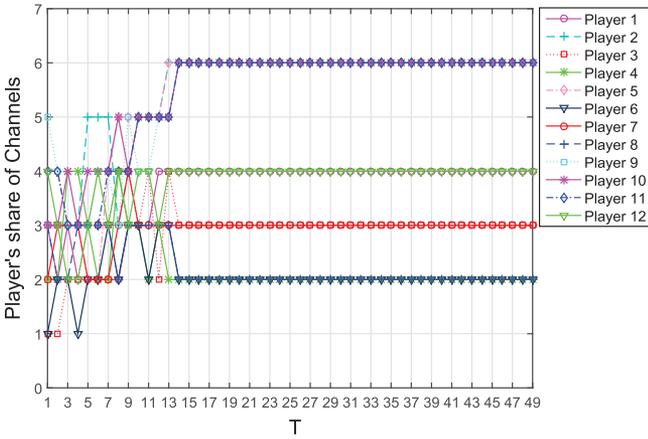
(b) Pareto optimal SPNE strategy $(x_1^0, x_{-1}^0)^T$ of a 12-player finite horizon game with varying number of periods (T). Here $M = 15$, and $\delta_i = 0.75 \; \forall i$.

Fig. 2. Pareto optimal SPNE of games with varying parameters.

on several factors– the number of players, N; the number of available channels, M; and the average discount factor of the players, δ. More precisely, we have,

$$T' \propto \frac{N}{M \cdot (1 - \delta)} \qquad (10)$$

Thus, T' – (i) *increases* as the number of player increases, (ii) *decreases* as more channels become available and (iii) *increases* as the discount factor of the players increases, i.e., as the players become more *patient* in waiting for the bargaining outcome. This trend can be verified from Figure 2(a) and Figure 2(b). Note that, for the game in Figure 2(a) the ratio on the R.H.S of (10) is lesser than that of the game in Figure 2(b). Note also that the game in Figure 2(a) converges earlier than the one in Figure 2(b). This corroborates (10). Therefore, based on the above discussion, to minimize the number of iterations required by Algorithm 5 to find T', T should be made proportional to $\frac{N}{M \cdot (1-\delta)}$.

2) How Long do Players Negotiate?: It is worth emphasizing here that the players do not actually "play" the bargaining

game over time periods to reach consensus. Specifically, the perspective adopted in the paper is the following- if the players were to play the spectrum bargaining game, then considering players to be selfish and rational, the outcome of the game would correspond to the SPNE solution presented in the paper. The SPNE solution itself (i.e., the equilibrium spectrum shares) can be computed by the players using Algorithm 3 (for the finite horizon case) or Algorithm 5 (for the infinite horizon case) in the very first period of the game, which implies that the bargaining always terminates in the first period. Furthermore, the players are also guaranteed to abide by the equilibrium strategies they obtain using the algorithms (in the first period), since by definition, the spectrum shares obtained would constitute a SPNE of the bargaining game from which there is no profitable deviation.

3) Equilibrium Coordination Among Players: Note that the spectrum bargaining game, in general, can have multiple equilibria. The question then becomes how would the players coordinate to an equilibrium. Such coordination can be attained practically by various techniques [12], such as, in specific, by having policies in a game (see, for example, the concept of *focal-point* introduced in [33] for equilibrium coordination among players), and, in general, by leveraging information structures available to players. For example, note that, for a given graph coloring procedure (used in deriving players' strategies) and a technique to find the pareto optimal point, an unique equilibrium exists (in the last period as well as for the SPNE of the overall game). Therefore, by having a policy on the use of optimization techniques among players to derive their equilibrium strategies, coordination among players can be achieved.

4) Last Mover Advantage: One might suspect that our spectrum bargaining game may exhibit a "last mover advantage", where a player making an offer in the last period may have an advantage over a player who gets to make an offer earlier. This effect diminishes as T increases, and vanishes in the infinite horizon case [4], [12]. Note that, in our spectrum bargaining game, the last mover advantage, and, in general, ordering of the players, practically will have a minimal impact. This is because the length of each bargaining period will be very short (in the order of time taken to exchange messages among nodes) allowing players to be able to afford a reasonable number of bargaining periods.

VI. SIMULATION RESULTS

In this section, we conduct simulations to study how the "self-gain" maximizing strategy of the players impact system wide performance. For simulations, we assume a noiseless, immobile radio network. The conflict graph, which represents interference constraints of the network, has been randomly generated with various graph densities. Also, in our simulations we assume that all players have an equal discount factor.

A. System Utility

If $\{x_i^t | 1 \le i \le N\}$ is accepted in period t, then we can define system utilities in terms of the payoffs of the players as follows.

- **Sum Utility:** This considers the total system utility regardless of fairness.

$$U_{sum} = \sum_{i=1}^{N} \mathbb{R}_i = \sum_{i=1}^{N} \delta_i^t |x_i^t| \qquad (11)$$

- **Minimum Utility:** This considers the utility of the player with the least payoff.

$$U_{min} = \min_{1 \le i \le N} \mathbb{R}_i = \min_{1 \le i \le N} \delta_i^t |x_i^t| \qquad (12)$$

- **Proportional Fairness based Utility** [25]:

$$U_{fair} = \sum_{i=1}^{N} log(\mathbb{R}_i) = \sum_{i=1}^{N} log(\delta_i^t |x_i^t|) \qquad (13)$$

To make it comparable to U_{min} and U_{sum}, we modify the fairness utility to:

$$U_{fair} = \left(\prod_{i=1}^{N} \mathbb{R}_i \right)^{1/N} = \left(\prod_{i=1}^{N} \delta_i^t |x_i^t| \right)^{1/N} \qquad (14)$$

Now, Algorithm 5 finds the infinite horizon SPNE strategy of P_1 in the *first* period ($t = 0$) of the game, which all rational players in P_{-1} will accept. Thus, the payoff of P_i ($i \in [1, N]$) will be $\mathbb{R}_i = \delta_i^0 |x_i^0| = |x_i^0|$. Based on this we simplify the metrics defined above as follows.

- U_{fair} : We use proportional fairness based system utility as defined in (14). Based on the above argument U_{fair} becomes,

$$U_{fair} = \sqrt[1/N]{\prod_{i=1}^{N} \delta_i^0 |x_i^0|} = \sqrt[1/N]{\prod_{i=1}^{N} |x_i^0|} \qquad (15)$$

Notice that, $U_{fair} = 0$ if there is any $|x_i^0| = 0$, $i \in [1, N]$. Thus, this metric will also help capture whether any node gets starved of channels.

- U_{mean} : We use mean utility instead of sum utility (11) in our simulations, so that all three utilities are within the same scale,

$$U_{mean} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{R}_i = \frac{1}{N} \sum_{i=1}^{N} \delta_i^0 |x_i^0| = \frac{1}{N} \sum_{i=1}^{N} |x_i^0| \qquad (16)$$

- U_{min} : We use minimum utility as defined in Equation (12). U_{min} becomes,

$$U_{min} = \min_{1 \le i \le N} \mathbb{R}_i = \min_{1 \le i \le N} |x_i^t| \qquad (17)$$

(a) Fairness

(b) Mean Utility

(c) Min Utility

Fig. 3. System Utility with varying number of available channels

B. Impact of the number of channels (M)

Figure 3 shows how the three utilities vary with the number of available channels, M. We consider a 8 node (player) network, with each player having a discount factor of 0.7. Figure 3(a) shows how U_{fair} varies with M. As can be seen from the figure, proportional fairness increases with the number of available channels. The graph also shows the impact of graph density on U_{fair}. For a given M (and N), U_{fair} decreases as graph density increases. Increasing graph density creates additional interference constraints. Thus, the average vertex degree in the conflict graph increases and each node tends to get lesser

number of channels. Therefore, U_{fair} scales inversely with graph density.

Figure 3(b) shows the average number of channels received by the nodes. As M increases, U_{mean} increases. Also, as graph density increases, for a given M and N, U_{mean} decreases due to the increase in average vertex degree. Figure 3(c) shows the minimum number of channels received by a node (U_{min}) as M increases. U_{min} increases as more channels become available. Since, U_{min} never falls below 1, no node is ever "starved" in the solution produced by our bargaining approach. This can also be noted from Figure 3(a). Moreover, the minimum value of U_{min} is 1 because we have considered that a player has to be offered at least 1 channel to make him accept an offer in the last period (Section IV). Generally speaking, our bargaining framework can be tailored to provide application specific minimum level of QoS.

C. Impact of the Number of Secondary Users (N)

Figure 4 shows how the three utilities degrade with increasing number of secondary nodes, N. The conflict graphs has been randomly generated with a graph density of 0.5. The discount factor of all players is 0.8. Figure 4(a) shows how proportional fairness based system utility, U_{fair}, vary with N. As N increases, U_{fair} decreases. This is because, as N increases (for a given graph density), more interference constraints are produced, thereby increasing average vertex degree in the conflict graph. As average vertex degree increases, and M remains fixed, each node tends to get lesser number of channels. Thus, U_{fair} is inversely proportional to the number of secondary users, N. As expected, for a given N and graph density, when more channels become available, U_{fair} increases.

Figure 4(b) plots the mean number of channels received by a node, U_{mean}, with varying number of secondary users, N. U_{mean} degrades with increasing N, due to the increase in average vertex degree in the conflict graph. As M increases, for a given N and graph density, each node on an average gets more channels. Note that, the rate of decrease of U_{mean} reduces with increasing N, i.e., U_{mean} tends to saturate around a minimum value for large N. This behavior can be more pronouncedly seen when the number of channels available is 10.

Figure 4(c) shows how the minimum number of channels received by any node, U_{min}, degrades with increasing N. As can be seen, U_{min} never falls below 1 (because of similar reasons explained for Figure 3(c)) regardless of the number of secondary users.

D. Impact of the Conflict Graph

In our bargaining model, we consider that the conflict graph is a given input. The conflict graph [17] itself can be based on either a distance-based criterion (i.e., the protocol model) or from signal strength values generated from RF propagation models (i.e., the physical model). The graph structure itself may depend not only the model used to generate the graph, but also on the parameters used in a specific model (such as, the specific SINR threshold used in the physical model and transmission power of nodes). For example, if the conflict graph is built
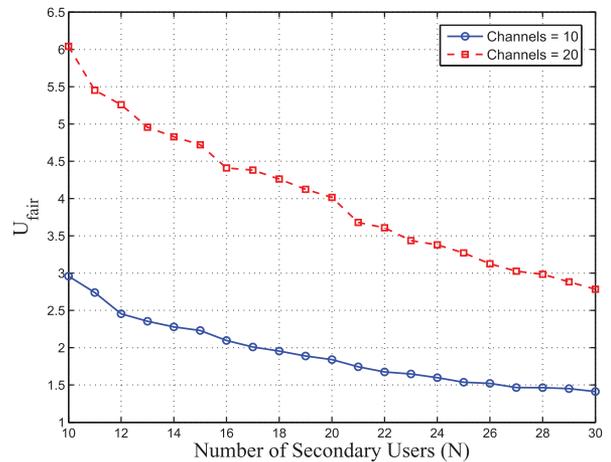
(a) Fairness
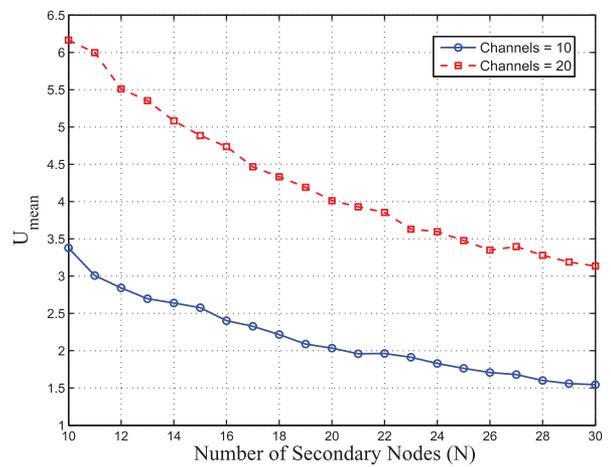
(b) Mean Utility
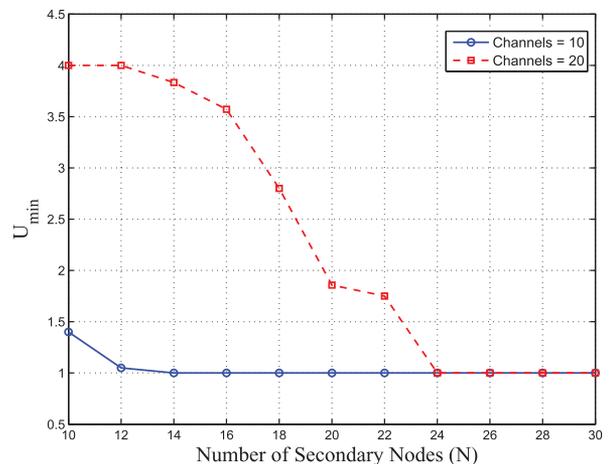
(c) Min Utility

Fig. 4. System Utility with varying number of secondary users.

using the physical model, more stringent SINR requirements will yield denser conflict graphs. Note that, the equilibrium bargaining solution may vary with the structure of the conflict graph. However, the algorithms presented in the paper will

work correctly (and yield the equilibrium bargaining solution) corresponding to the given conflict graph regardless of the model and parameters used to build the conflict graph.

We will now study how conflict graph density (which reflects various system parameters as mentioned above) affect spectrum sharing by the secondary users by studying the three system utilities. Figure 5 shows how the three utilities degrade as conflict graph density increases. The number of channels available, M, was set to 15 and the discount factor of all players in 0.8. From Figure 5(a) it can be clearly seen that U_{fair} degrades with increasing graph density for a fixed number of secondary users, N. This is again because the average vertex degree of the conflict graph increases with increasing graph density. For a given graph density, as N decreases, average vertex degree decreases, and thus U_{fair} increases for fixed M. Note that, when graph density is 1, we have a complete graph on N vertices, i.e, all nodes are within the interference range of each other. In this case, when N is 15, all nodes get a channel each (recall $M = 15$) and thus U_{fair} becomes 1.

Figure 5(b) plots U_{mean} with varying graph density. As can be seen, U_{mean} decreases with increasing graph density due to the increase in average vertex degree. Again, when N is 15 with a graph density of 1, U_{mean} becomes 1. Figure 5(c) shows the degradation of U_{min} with increasing graph density.

Note that the spectrum reusing capability of the network deteriorates more rapidly with increasing conflict graph density for a fixed number of secondaries than it degrades with increasing number of secondary users for a fixed graph density. This can be clearly seen from the graphs of U_{fair} and U_{mean} in Figure 4 and Figure 5.

E. Impact of Discount Factor

In Figure 6, the y-axis corresponds to the number of periods, T, of a finite horizon bargaining game of N players at which Algorithm 5 finds $|(x_1^0, x_{-1}^0)_j^T| = |(x_1^0, x_{-1}^0)_j^{T-1}|, \forall j \in [1, N]$, i.e., the period T at which the SPNE strategy of P_1 converges, thus corresponding to his strategy in the infinite horizon game, $(x_1^0, x_{-1}^0)^\infty$. The x-axis corresponds to varying discount factor, δ, of the players. Conflict graph density has been taken to be 0.6 and the number of channels available, M, is 14. As can be seen from the figure, T scales inversely with $1 - \delta$. This is because, as the discount factor of the players increases, the players become more *patient* in waiting for the bargaining outcome. When δ is 1, the players can wait infinitely long for the bargaining outcome. Moreover, it can also be noted that T increases as the ratio of $N : M$ increases. This can be clearly seen, since, T for any given discount factor is least for $\frac{N}{M} = \frac{10}{14}$, increases when $\frac{N}{M} = \frac{20}{14}$ and highest when $\frac{N}{M} = \frac{40}{14}$. These observations corroborate (10).

F. Price of Anarchy

We will now compare the solution of our proposed spectrum bargaining game (in which players seek to optimize their *own utilities*) with a channel allocation mechanism which seeks to optimize the *overall system utility*. Such an analysis will illustrate the price-of-anarchy [12] of the spectrum bargaining
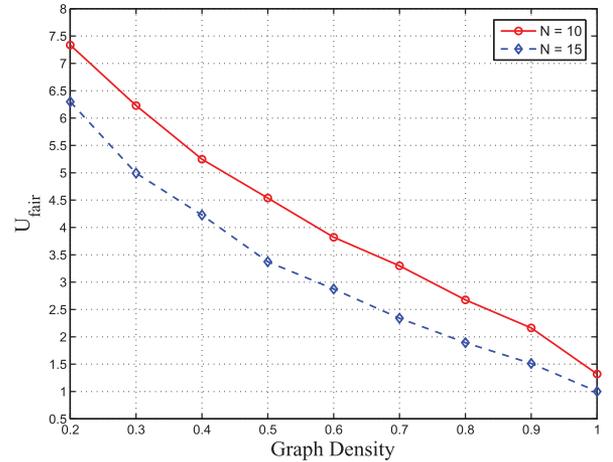
(a) Fairness
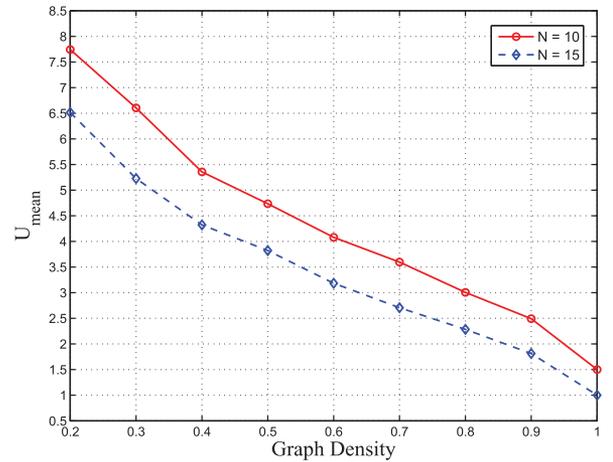
(b) Mean Utility
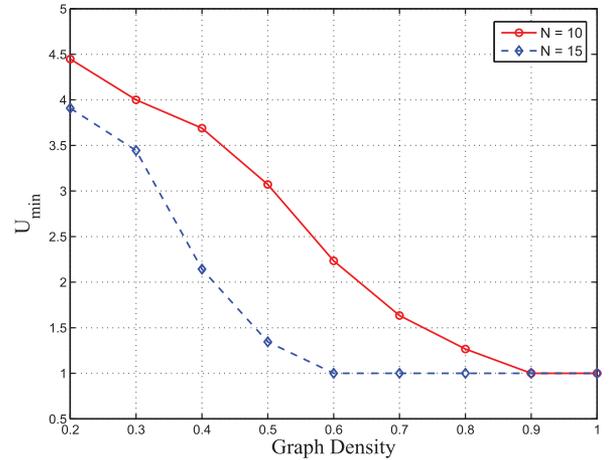
(c) Min Utility

Fig. 5. System Utility with varying conflict graph density.

game, which measures how the overall system utility degrades due to selfish behavior of its agents. Specifically, we will study how system utility is affected by two different approaches– (i) our spectrum bargaining approach where users want to maximize their individual utilities, and (ii) graph multi-coloring
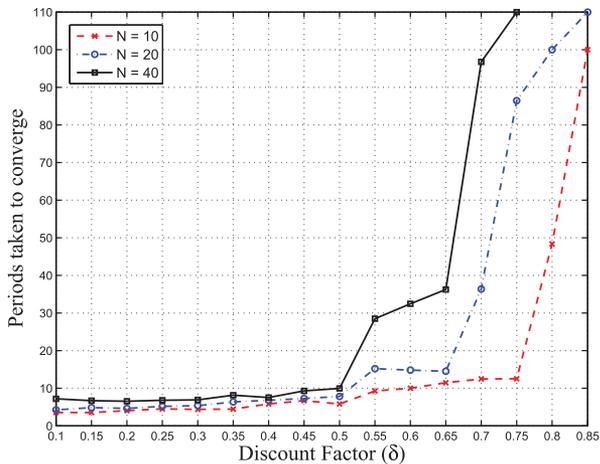
Fig. 6. Convergence period T of first period offerer's SPNE strategy with varying discount factor of players.
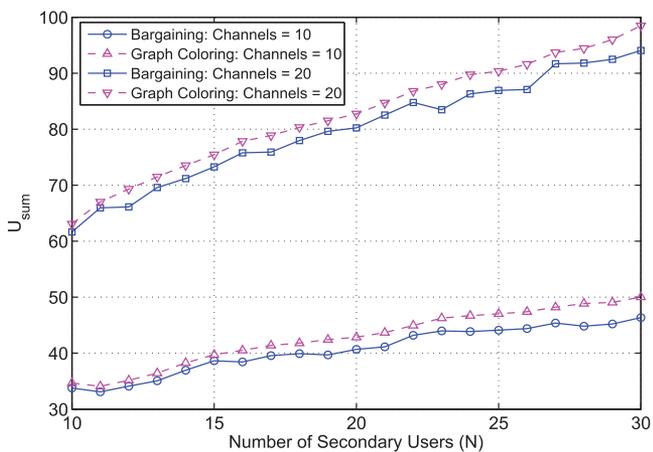
Fig. 7. Price of Anarchy

approach (GMC) which seeks to maximize overall system utility.

For completeness, we will briefly review the GMC approach. In [29], [40], the authors have shown that by mapping each channel into a color, the problem of channel allocation to maximize system utility can be modeled as a graph multi-coloring (GMC) problem. In a GMC problem, the objective is to color each vertex of the graph using a number of colors from a set of available colors (analogous to the set of available channels C in our bargaining model), and find the color assignment that maximizes the sum of the number of colors assigned to all the vertices, i.e., the sum utility. The coloring is constrained by the fact that if an edge exists between any two distinct vertices, they can't be colored with the same color. Efficient algorithms for channel allocating via the GMC approach have been proposed by the authors in [40] based on which we implement the GMC based channel allocation scheme.

Figure 7 shows how U_{sum} varies with the number of secondary users for our bargaining based solution approach and the GMC scheme (referred to as graph coloring in Figure 7). The conflict graph has been randomly generated with a graph density of 0.5. The discount factor of all players is 0.8. As can

be seen from the figure, the U_{sum} is slightly larger when graph coloring is used. This is because the GMC approach explicitly seeks to maximize the sum utility of the overall network, while in the bargaining approach, players seek to maximize their own utilities (which need not maximize the overall utility). Notice, however, that the difference between the system utilities achieved by the two approaches is not much, implying that our bargaining based spectrum sharing approach maximizes individual utilities while achieving reasonably good system wide utility.

VII. CONCLUSIONS

This paper models the problem of dynamic spectrum access by a set of cognitive radio enabled nodes as a bargaining game where the nodes bargain among themselves in a distributed manner to agree upon a sharing rule of the channels. First, the paper explores the finite horizon version of the bargaining game and presents computationally efficient algorithms to find the Pareto optimal SPNE strategy of the player making the offer in the first period of the game. This is a strategy, such that, neither can the player making the offer increase his utility by making any other offer, nor can the players receiving the offer gain in any subsequent period by rejecting this offer. Next, we extend the results from the finite horizon game to find the Pareto optimal SPNE strategies of the infinite horizon game. Finally, using simulations we study the how the selfish strategies of the players affect system wide performance.

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