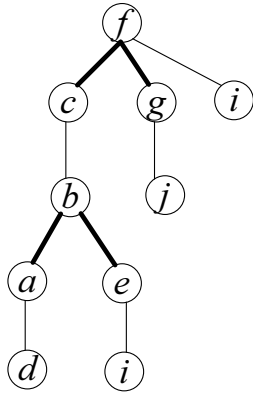


評分標準

有關第 2 題裡頭建造 BFS-like alternating tree 時，不少同學抄襲到同一份錯誤的答案，情況如下：



在這個圖裡頭，由於 f 已經是 matched node，不可當 root。注意：alternating tree 的 root 必須是 single node，不可以是 matched node。所以只要畫出此圖，一律扣 10 分。

對於第 3 題，同學大部分抄襲到二種版本。

第一種版本如下：

Let F be a Boolean formula in CNF. For each literal in F , we will make a vertex in the graph, i.e. $(x_1+x_2+x_3)(x_1+x_2+x_3)$ has 6 vertices.

Let k be the number of clause in F . We will connect each vertex to all of the other vertices that are logically compatible except for the ones that are in the same clause. Now if we have a satisfying vertices will all be connected to one another. Thus we can use clique to solve 3SAT. So clique problem is NP-complete.

首先，「If we have a satisfying vertices will all be connected to one another.」這真是一句文法錯誤到令人頭皮發毛的句子。(1) If 開頭，後面總要有個「則...」，完全沒看到。(2) will 的主詞是啥？是「if we have a satisfying vertices」嗎？(3) 「a satisfying vertices」是一個可怕的名詞--前面有冠詞 a，後面的名詞 vertices 居然是複數。最後，什麼是「satisfying vertices」？從未聽聞。如果是同學自己發明的專有名詞，必須告訴我們「satisfying vertices」的定義。但最糟糕的是：「Thus we can use clique to solve 3SAT」....這話很怪，完全沒告訴我們如何用 clique 解 3SAT。可能同學是抄襲

<http://www.cs.berkeley.edu/~isabelle/cs302/ps7-solutions.pdf>。但這裡頭的答案只是「提示」，而非「詳解」。同學必須告訴我們如何將 clique 轉成 3SAT 的解（哪些 Boolean 變數的值是 1，哪些 Boolean 變數的值是 0）。所以只要是寫這個答案，一律扣 10 分。

第二種版本是抄襲網路的答案，如下：

Given a 3-CNF formula F of m clauses over n variables, we construct a graph as follows. For

each clause c of F we create one node for every assignment to variables in c that satisfies c .

E.g., say

$F = (x_1 \text{ OR } x_2 \text{ OR } \text{not}(x_4)) \text{ AND } (\text{not}(x_3) \text{ OR } x_4) \text{ AND } (\text{not}(x_2) \text{ OR } \text{not}(x_3)) \text{ AND } \dots$

Then in this case we would create nodes like this:

$(x_1=0, x_2=0, x_4=0) (x_3=0, x_4=0) (x_2=0, x_3=0) \dots$
 $(x_1=0, x_2=1, x_4=0) (x_3=0, x_4=1) (x_2=0, x_3=1)$
 $(x_1=0, x_2=1, x_4=1) (x_3=1, x_4=1) (x_2=1, x_3=0)$
 \dots

Then we put an edge between two nodes if the partial assignments are consistent. Note: max possible clique size is m . And, if the 3-SAT problem does have a satisfying assignment, then in fact there IS an m -clique. Claim is this is true in the other direction too. If the graph has an m -clique, then there is a satisfying assignment: namely, just read off the assignment given in the nodes of the clique. So, this graph has a clique of size m iff F was satisfiable. Also, our reduction is poly time since the total size of graph is at most quadratic in size of formula ($O(m)$ nodes, $O(m^2)$ edges). Therefore Max-Clique is NP-complete.

這個版本的答案至少沒有什麼英文文法的問題。但最大問題出在：「 $F = (x_1 \text{ OR } x_2 \text{ OR } \text{not}(x_4)) \text{ AND } (\text{not}(x_3) \text{ OR } x_4) \text{ AND } (\text{not}(x_2) \text{ OR } \text{not}(x_3)) \text{ AND } \dots$ 」這個 Boolean formula 裡頭，括弧的裡頭 Boolean 變數居然可以只有 2 個，這並非 3SAT 的型式。翻開課本第 1082 頁，按 3SAT 的定義，括弧的裡頭 Boolean 變數必須剛好（或至少）3 個 Boolean 變數。我再查一下網站

<https://www.cs.cmu.edu/afs/cs/academic/class/15451-f05/www/lectures/lect1108.txt>，這裡頭提到 3SAT 的定義如下：「3-SAT: Given: a CNF formula (AND of ORs) over n variables x_1, \dots, x_n , where **each clause has at most 3 variables** in it. $(x_1 \text{ OR } x_2 \text{ OR } \text{not}(x_3)) \text{ AND } (\text{not}(x_2) \text{ OR } x_3) \text{ AND } (x_1 \text{ OR } x_3) \text{ AND } \dots$ Goal: find an assignment to the variables that satisfies the formula if one exists.」

這網站說，在 3SAT 的問題裡頭，括弧裡頭的 Boolean 變數最多是 3 個。這個 3SAT 的定義是錯的，因為如果我們允許括弧裡頭的 Boolean 變數可以只有 2 個，那麼這種變形的 SAT problem 將不再是 NP-complete，而是落在 P 裡頭（參考課本 1049 頁或者 <http://en.wikipedia.org/wiki/2-satisfiability>）。所以只要是抄襲這個版本，一律扣 5 分。

正確的證明在課本第 1087~1089 頁。